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INDIRECT PARAMETER ESTIMATION OF INDUCTION MOTOR BASED ON TIME DOMAIN ANALYSIS

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Abstract - *The paper presents a new method of estimating the parameters of squirrel cage induction motor. By using a non-parametric model, estimated through in a dc step response test, the connection between its coefficients and those of the equivalent phase circuit is obtained. Moreover a new method of establishing the dependence of parameters in terms of saturation level is coming out.*

Key Words: ***modelling, identification, estimation, vector-controlled induction motor drives***

1. INTRODUCTION

In most industrial electrical drives the induction machine is preferred over the other rotating machines because of its rugged construction. Fast development of microprocessor technology enables application of complex control algorithms, particularly the vector control techniques, in order to achieve high dynamic performance from induction machine in terms of torque. Vector controller can be decomposed into feedback controller (performance controller) and feedforward controller (decoupling network). Both of them have the common feature that they are motor parameter dependent. Therefore, the vector control is only effective as long as the values of the motor parameters in the controller are in agreement with the actual motor parameters. On the other hand motor parameters change with temperature, frequency and magnetic saturation.

The mathematical model of an induction motor is a set of non-linear differential equations, which excludes the use of a simple and effective on-line identification method [1]. Hence the problem of identification only of rotor resistance is most often discussed. Usually, the remainder parameters are considered constant and are estimated on the basis of an off-line parameter identification technique.

In this paper we present a new method to estimate the parameters of the squirrel cage induction motor, based on digital processing of the step response of the induction motor configured into a special topology. The tests were performed at different current levels allowing the determination of all parameters dependency in terms of the magnetising current. Thus, the effect of the saturation may be taken into account as a possible alternative to the motor model, using the look-up table technique or polynomial function approximation, enabling to implement an adaptive control with "gain scheduling".

2. PARAMETERS IDENTIFICATION PRINCIPLE

We consider the voltage controlled induction motor whose dynamics can be represented by six equivalent state models. If, in a fixed stator reference frame (w_g=0), space vectors $\underline{i}_s$ and $\underline{Y}_s$ are chosen as state variables the following model is obtained:

$$\begin{cases} \dot{X} = AX + BU \\ Y = CX \end{cases} \quad (1)$$

where:

$$X = \left[\underline{i}_s, \underline{\Psi}_s\right]^T; U = \underline{u}_s; Y = \underline{i}_s \quad (2)$$

$$A = \begin{bmatrix} -\frac{1}{s}\left(\frac{1}{T_s} + \frac{1}{T_r}\right) + jw_r & \frac{1}{sL_s}\left(\frac{1}{T_r} - jw_r\right) \\ R_s & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{1}{sL_s} & 1 \end{bmatrix}^T; \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad (3)$$

and:

$s = 1 - \frac{L_m^2}{L_s L_r}$ - global leakage factor ;

$T_s = \frac{L_s}{R_s}; T_r = \frac{L_r}{R_r}$; - stator/rotor time constants

$\underline{i}_s, \underline{u}_s, \underline{y}_s$ - stator current/voltage/flux space vectors

w_r - rotor angular speed.

The dynamic behaviour of the system depends, both quantitative and qualitative, upon complex eigenvalues [2],obtained as solutions of the characteristic polynomial:

$$l_{1,2} = -\frac{1}{2}\frac{1}{s}\left(\frac{1}{T_s} + \frac{1}{T_r}\right) - jw_r \pm \frac{1}{2}\sqrt{\left[\frac{1}{s}\left(\frac{1}{T_s} + \frac{1}{T_r}\right) - jw_r\right]^2 - \frac{4}{sT_s}\left(\frac{1}{T_r} - jw_r\right)} \quad (4)$$

If a modal representation of the transition matrix is used it is easy to obtain the analytical solution of the system equations. In such a way the general solution will be a linear combination of solutions of the first-order differential equations and the general matrix form will be avoided [3]:

$$\begin{cases} X(t) = Se^{\Lambda t}S^{-1}X(0) + Se^{\Lambda t}\int_0^t e^{-\Lambda t}S^{-1}BU(t)dt \\ Y(t) = CSe^{\Lambda t}S^{-1}X(0) + CSe^{\Lambda t}\int_0^t e^{-\Lambda t}S^{-1}BU(t)dt \end{cases} \quad (5)$$

where:

$$\Lambda = \begin{bmatrix} l_1 & 0 \\ 0 & l_2 \end{bmatrix}; \qquad S = \begin{bmatrix} 1 & 1 \\ -\dfrac{R_s}{l_1} & -\dfrac{R_s}{l_2} \end{bmatrix}$$

$$S^{-1} = \frac{l_1 l_2}{l_2 - l_1}\frac{1}{R_s}\begin{bmatrix} -\dfrac{R_s}{l_2} & -1 \\ \dfrac{R_s}{l_1} & 1 \end{bmatrix} \quad (6)$$

For the step response (U (τ)=U$_0$) the solution (5) will be

$$Y(t) = CSe^{\Lambda t}S^{-1}X(0) + CS\Lambda^{-1}(e^{\Lambda t} - I_2)S^{-1}BU_0 \quad (7)$$

and the following is obtained:

$$\underline{i}_s(t) = \frac{1}{l_2 - l_1}\left[l_2 e^{l_2 t} - l_1 e^{l_1 t}\right]\underline{i}_s(0) + \frac{l_1 l_2}{l_2 - l_1}\frac{1}{R_s}\left[e^{l_2 t} - e^{l_1 t}\right]\underline{\Psi}_s(0)$$
$$+\frac{\underline{u}_{0s}}{R_s}\left[1 + \left[\frac{1}{l_2 - l_1}\left(l_1 + \frac{1}{sT_s}\right)\right]e^{l_1 t} + \left[\frac{1}{l_1 - l_2}\left(l_2 + \frac{1}{sT_s}\right)\right]e^{l_2 t}\right] \quad (8)$$

The general analytical form of the step response, (8), can be used to estimate the physical parameters for the phase equivalent circuit. In this order the squirrel cage induction motor will be connected as shown in Fig. 1.

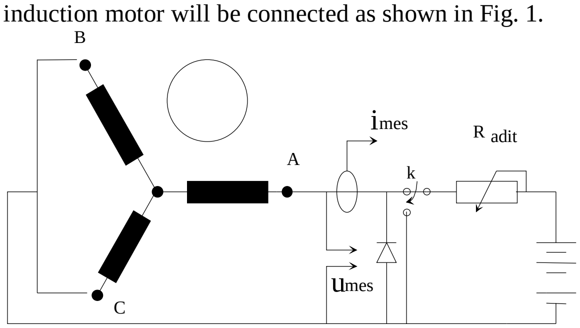

Fig.1. *Experimental set-up*

This configuration is characterised by the following relations between phase values (voltages, currents) and measured values:

$$i_B = i_C = -\frac{i_A}{2} = -\frac{i_{mes}}{2};$$
$$u_A = \frac{2}{3}u_{mes}; \quad u_B = u_C = -\frac{1}{3}u_{mes} \quad (9)$$

By using the space vectors power invariant definition the d-q axis components have the following values:

$$i_{sd} = \sqrt{\frac{3}{2}}i_{mes}; \; i_{sq} = 0; \; u_{sd} = \sqrt{\frac{2}{3}}u_{mes}; \; u_{sq} = 0 \quad (10)$$

The motor is supplied from a dc voltage source, as shown in fig.1, and the current that flows into machine stator windings is imposed by the means of a variable resistor R$_{adit}$. When the switch k is commuted the electromagnetic transient mode is started, and the current i_{mes} is recorded through an A/D converter from the multifunction testbench [4]. The recorded data represents the step response of the motor at a negative voltage step, -u_{mes}, ($\underline{u}_{0s} = 0$). Thus the step response (8) becomes:

$$\underline{i}_s(t) = i_{sd}(t) = \frac{1}{l_2 - l_1}\left[l_2 e^{l_2 t} - l_1 e^{l_1 t}\right]i_{sd}(0) + \frac{l_1 l_2}{l_2 - l_1}\frac{1}{R_s}\left[e^{l_2 t} - e^{l_1 t}\right]\Psi_{sd}(0) \quad (11)$$

In addition, under steady-state conditions (t=0), the initial values of the state variables are related by:

$$\Psi_{sd}(0) = L_s i_{sd}(0) \quad (12)$$

By replacing relation (12) in (11) the following non-parametric model is obtained:

$$i_{sd}(t) = \frac{l_1 i_{sd}(0)}{l_1 - l_2}\left[1 + l_2 T_s\right]e^{l_1 t} + \frac{l_2 i_{sd}(0)}{l_2 - l_1}\left[1 + l_1 T_s\right]e^{l_2 t} = C_1 e^{l_1 t} + C_2 e^{l_2 t} \quad (13)$$

Between the coefficients of the non-parametric model (13), more precisely C_1, C_2, l_1 and l_2, and physical parameters of the induction motor, exists a rigorous correlation. Thus, remembering that in such an experiment the rotor is at standstill (w_r=0), the expressions of the eigenvalues (4) become:

$$\begin{cases} l_1 = -\dfrac{1}{2}\dfrac{1}{s}\left(\dfrac{1}{T_s} + \dfrac{1}{T_r}\right) + \dfrac{1}{2}\sqrt{\left[\dfrac{1}{s}\left(\dfrac{1}{T_s} + \dfrac{1}{T_r}\right)\right]^2 - \dfrac{4}{sT_sT_r}} \\ l_2 = -\dfrac{1}{2}\dfrac{1}{s}\left(\dfrac{1}{T_s} + \dfrac{1}{T_r}\right) - \dfrac{1}{2}\sqrt{\left[\dfrac{1}{s}\left(\dfrac{1}{T_s} + \dfrac{1}{T_r}\right)\right]^2 - \dfrac{4}{sT_sT_r}} \end{cases} \quad (14)$$

Based on equation (14) it follows:

$$\begin{cases} l_1 + l_2 = -\dfrac{1}{s}\left(\dfrac{1}{T_s} + \dfrac{1}{T_r}\right) \\ l_1 l_2 = \dfrac{1}{sT_sT_r} \end{cases} \quad (15)$$

and also considering (13) we obtain:

$$\begin{cases} T_s \overset{def}{=} \dfrac{L_s}{R_s} = -\dfrac{C_2 l_1 + C_1 l_2}{l_1 l_2 (C_1 + C_2)} \\ T_r \overset{def}{=} \dfrac{L_r}{R_r} = -\dfrac{C_1 l_1 + C_2 l_2}{l_1 l_2 (C_1 + C_2)} \\ s \overset{def}{=} 1 - \dfrac{L_m^2}{L_s L_r} = \dfrac{l_1 l_2 (C_1 + C_2)^2}{(C_1 l_1 + C_2 l_2)(C_2 l_1 + C_1 l_2)} \end{cases} \quad (16)$$

If the stator resistance R_s is measured

$$R_s = \frac{u_{sd}}{i_{sd}} = \frac{2}{3}\frac{u_{mas}}{i_{mas}} \quad (17)$$

it is obvious that

$$L_s = T_s R_s \quad (18)$$

is the last parameter we can directly determine.

It is known that the induction machine is not a completely observable system [5,6], and that several motors with same impedance but varied electromagnetic parameters may exists. The simplest measuring parameter is the stator leakage inductance L_{ss}, which can be estimated by several methods [7]. If the stator leakage inductance is estimated, then the remainder parameters can be obtained from:

$$L_m = L_s - L_{ss}; L_r = \frac{L_m^2}{(1-s)L_s}; L_{sr} = L_r - L_m;\ R_r = \frac{L_r}{T_r} \quad (19)$$

The decay current values must be approximated with a function like:

$$F(t, l_1, l_2, C_1, C_2) = C_1 e^{l_1 t} + C_2 e^{l_2 t} \quad (20)$$

Due to non-linear character, the fitting curve is obtained from recorded values by a non-linear regression method. The Levenberg-Marquardt algorithm, most used in practice due to high rate of convergence [8], minimise the following cost function:

$$V(q) = \frac{1}{2}\sum_{i=1}^{N} e_i^2 = \frac{1}{2}\sum_{i=1}^{N}\left(i_{sd}(i) - F(t(i), q)\right)^2 \quad (21)$$

Theoretically the stator leakage inductance may be approximated by the homopolar inductance, which is obtained from voltage, current and active power measured in an open delta topology. In practice we notice a significant difference between estimated values of stator resistance R_s obtained from the above method and values obtained from the volt/ampere method, aspect pointed out also in [9]. In our opinion, the greater value of stator resistance estimated in ac test is due to core loses and to space harmonics caused by non-sinusoidal distribution of stator windings. Therefore, to avoid this effect the test was performed without rotor using the same hardware as shown in Fig. 1. In transient mode the circuit is described by the following differential equation:

$$\frac{di_{s0}(t)}{dt} = -\frac{R_s}{L_{ss}} i_{s0}(t) + \frac{1}{L_{ss}} u_{s0}(t) \quad (22)$$

with eigenvalue:

$$l = -\frac{R_s}{L_{ss}} \quad (23)$$

Thus the step response is:

$$i_{s0}(t) = e^{lt} i_{s0}(0) + \frac{1}{l}\left(e^{lt} - 1\right)\frac{1}{L_{ss}} u_{s0} \quad (24)$$

and for a negative step (u_{s0}=0) on obtains:

$$i_{s0}(t) = e^{lt} i_{s0}(0) = C e^{lt} \quad (25)$$

In order to obtain the λ parameter the Levenberg-Marqardt algorithm is used again and stator leakage inductance is computed from:

$$L_{ss} = -\frac{R_s}{l} \quad (26)$$

3. EXPERIMENTAL RESULTS

The trials were performed on a 0,55KW squirrel cage induction motor whose rated values are presented in appendix. As an example, the recorded values of the stator current and the result obtained from data processing are presented in Fig.2. The sampling frequency is f_e=8 KHz and the steady state current is 2.073A.

For this case the used non linear regression method yields the following non-parametric model:

$$i_{fit}(t) = 0.7997 e^{-11.0045t} + 1.2684 e^{-261.32t} \quad (27)$$

The fitted curve is presented comparatively with acquired one in Fig.3.

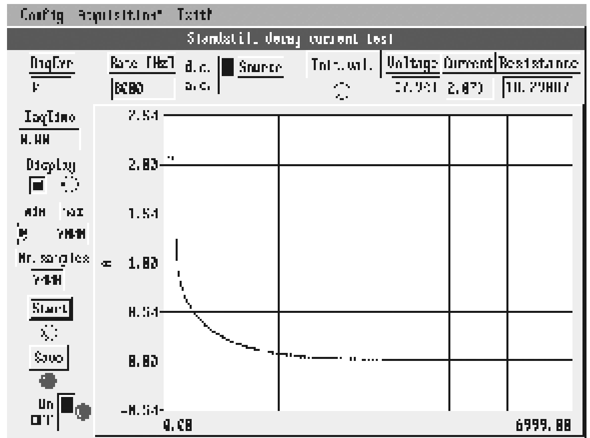

Fig.2. *Acquired values*

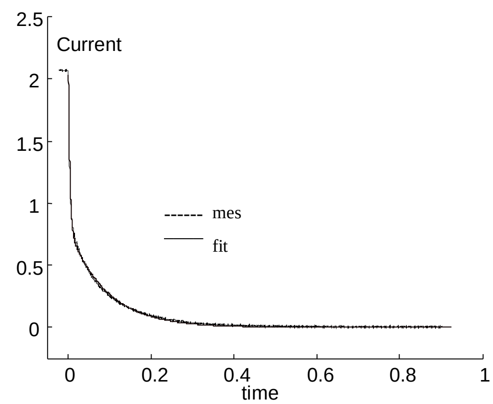

Fig.3 *Acquired and fitted curves of decay current*

With chosen sampling frequency the estimated values (i_{fit}) matches the measured values with less ±1% error.

In order to avoid noise measuring errors a number of N=75 samples of voltage and current, in steady state, was acquired and the stator resistance is obtained as a mean value as shown bellow:

$$R_s = \frac{2}{3}\frac{\frac{1}{N}\sum_{i=1}^{N} u_{mes}(i)}{\frac{1}{N}\sum_{i=1}^{N} i_{mes}(i)} \quad (28)$$

In this stage from (27) and (28) the four parameters defined in (16) and (19) can be computed.

Based on (25), (26) the stator leakage inductance was computed, and obtained values are presented in Fig. 4.

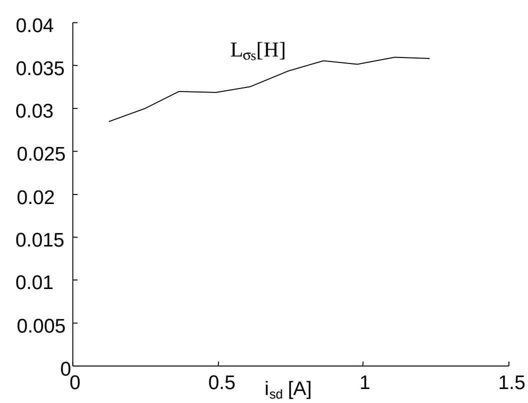

Fig. 4. *Experimental values of leakage inductance*

In order to determine the other parameters defined in (19) a $L_{ss} = 0.0348\,\mathbf{H}$ value of stator leakage inductance was adopted.

Trials were performed at different values of the steady state current, allowing establishing the dependence of parameters in terms of magnetizing current.

The results of the described methodology in terms of i_{sd} are presented in Fig.5 - Fig. 11.

We notice the variation of mutual inductance in accordance with theoretical predictions. However unexpected is the rotor resistance variation, thermal effects being excluded due to short duration of transitory regime (< 1s).

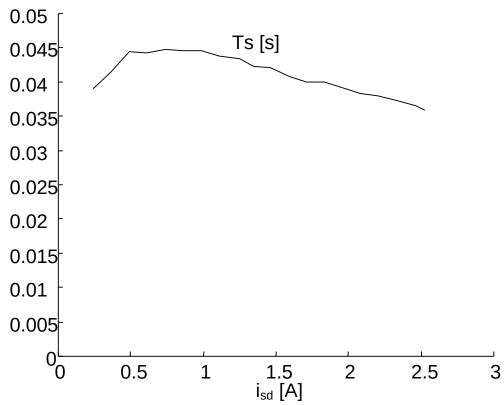

Fig. 5. *Stator time constant*

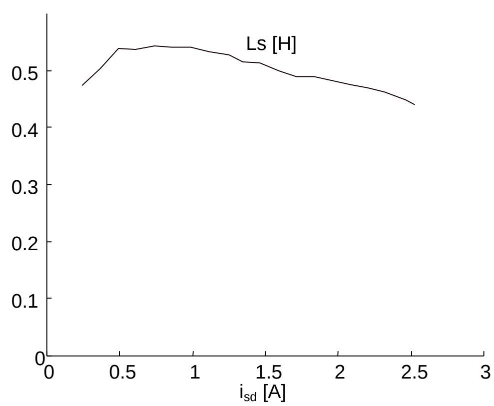

Fig.6. *Stator inductance*

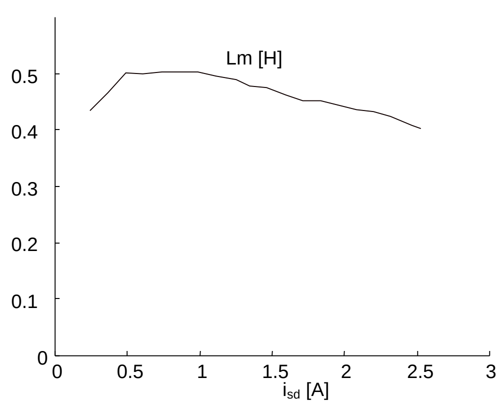

Fig.7. *Mutual inductance*

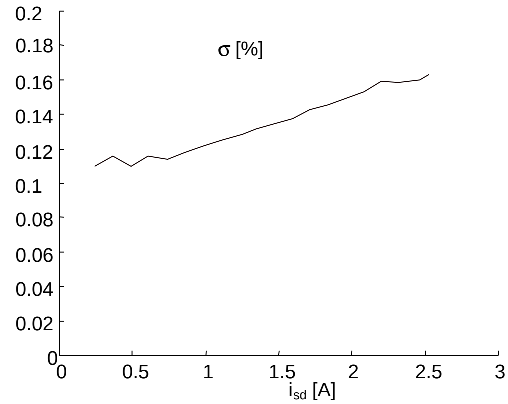

Fig.8. *Global leakage factor*

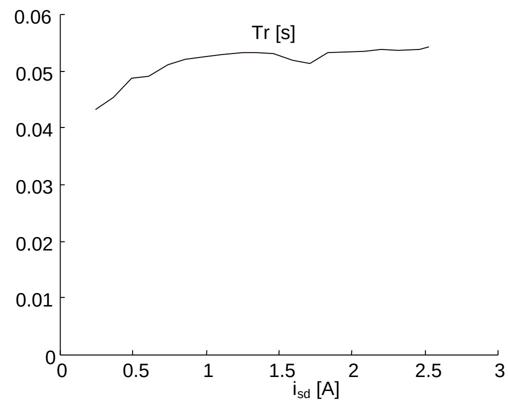

Fig.9. *Rotor time constant*

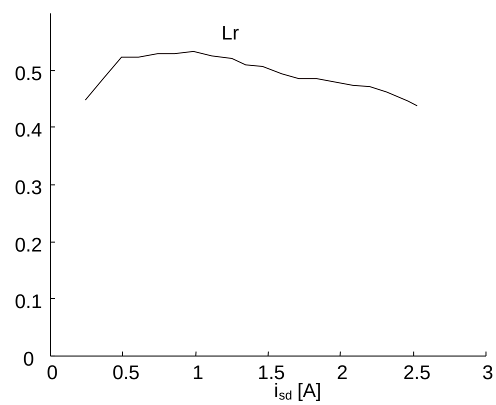

Fig.10. *Rotor inductance*

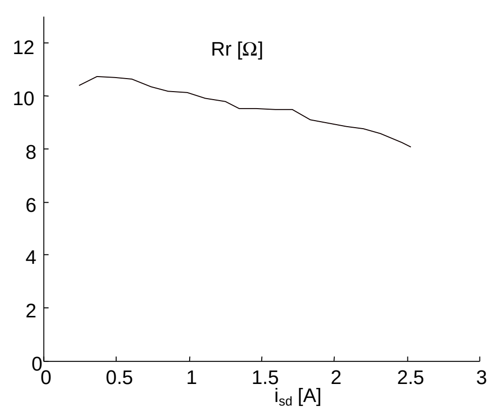

Fig.11. *Rotor resistance*

If we try to analyse the physical phenomena at rotor slots level, we think that reduction of the magnetic permeability of the iron paths as a consequence of the magnetic saturation determines the increasing of teeth reluctance and the decreasing of the rotor leakage inductance with immediate effect in a current "spreading" in rotor bars. This current "spreading" is a possible explication for rotor resistance decreasing with increasing of magnetising current.

4. METHOD VALIDATION

In order to validate our results the acquired values was also processed upon a classical method like standstill decay current test [7]:

$$\Psi_m(0) = R_s \int_0^{\infty} i_{sd}(t)dt - L_{ss} \cdot i_{sd}(0) \tag{29}$$

$$L_m(i_{sd}) = \frac{\Psi_m(0)}{i_{sd}(0)} \tag{30}$$

As shown in Fig. 12 the obtained results are very closed.

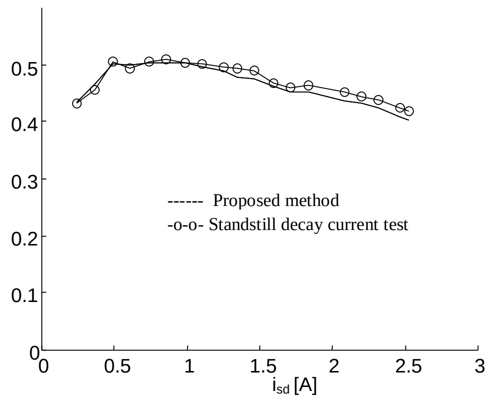

Fig.12. *Comparative results for L_m*

But, in contrast with standstill decay current method, the proposed method offer the means to estimate the rotor parameters.

5. CONCLUSIONS

Advanced on-line parameter estimation algorithms suppose an *a priori* knowledge of the controlled system. In this order preliminary trials may be performed to establish some system characteristics. The present contribution to induction motor parameter estimation consists of a method based on current recording as a voltage step response and off-line parameter estimation from a curve fitting technique. In such a way the parameters variations related to magnetising current might be studied.

Determination of saturated parameter values based on numerical field methods may be difficult due unpredictable technological influences. Thus the experimental results obtained by this method can have a benefit in order to improve the machines design

Moreover, in order to simulate an induction motor drive system, this method offers the possibility of modeling the saturation, by means of experimentally established magnetizing curve, or parameter variation [10].

This estimating method does not offer the possibility to evaluate the losses that occur in the motor at normal operation, losses, which can significantly alter the performances of high precision, drive systems [11]

The presented method has the advantage to perform trials at low power and has a better precision as compared with standard methods.

6. APPENDIX

Main parameters of the induction machine

Δ/Y	220/380V	3.05/1.76A
P=0.55 kW		cosφ=0.687
n_n=930 rev/min		f_n=50 Hz

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