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AN ANALYTICAL APPROACH FOR SOME FAMILIES OF VOLTAGE DISTRIBUTION DIODES NETWORKS EQUATIONS

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Abstract: *The genesis of an analytical closed form solution for some families of nonlinear diode circuit equations is studied in some detail, and an example is presented theoretically and numerically. Namely, using the Special Trans Function Theory (STFT) the analytical closed-form solution to the nonlinear circuit signals is obtained. The structure of derivation, appropriate proofs and numerical results for a choice example confirm the validity of STFT. The conditions for the existence of the exact solutions are discussed.*

Key Words: ***The Special Trans Function Theory, Analytical closed form solution, Voltage distributions diodes networks***

1. INTRODUCTION

The Special Trans Functions Theory -STFT ([1]-[11]) for obtaining meaningful results from analytical study of the nonlinear functional equations is an important one. The present paper researches possibilities for an analytical closed form solution of some families of nonlinear circuit equations (voltage distributions diodes networks equation) because the former methods cannot express the solutions by an analytical closed form.

Consequently, the subject of the theoretical analysis presented here is some families of nonlinear functional equations of the form [1]

$$\sum_{n=0}^{N}\alpha_n Y^n = \sum_{k=1}^{K}\beta_k Y^k e^{-kY} + \delta . \tag{1}$$

The parameters have the following mining: N is the maximum of degree of nonlinearity to the researched functional equation; K is the maximum of exponential terms of nonlinear functional equation; α_n, β_k are functional parameters inherent to the real nonlinear circuit described by equation (1). Analytical closed form solutions to the nonlinear functional equations (1), obtained by application of the STFT, are a new special tran functions, defined as

$$Y = trans_{\text{"IDprocess"}}(\alpha_n, \beta_k) \qquad n = 1,\ldots,\ldots N; \quad k-1,\ldots,\ldots K \tag{2}$$

where "IDprocess" denotes an important characteristic of the studied nonlinear circuit, or a significant parameter to the nonlinear circuit. The genesis of analytical solution, as new special function in the STFT, is not complicated, and based on the fact that the transcendental equation (1) can be identified with a suitable partial differential equation. Consequently, the transcendental equation (1) can be identified with a partial differential equation of type ([1])

$$\sum_{n=0}^{N}\alpha_n \frac{\partial^{ni} F}{\partial x^n} = \sum_{k=1}^{K}\beta_k \frac{\partial^k F}{\partial x^k} + \delta F(x). \tag{3}$$

This equation for identification (EQID) has the analytical, unique, closed form solution and particular solution which has the appropriate asymptototic nature in comparison with the unique analytical closed form solution. Namely, the equation for identification (3) validity implies the particular (as an asymptotic) unique solution existence in the exponential form

$$F_{as}(x,\alpha_1,\ldots\alpha_{N,}\beta_1,\ldots,\beta_K) = F_{ao}\exp(Y(\alpha_1,\ldots\alpha_{N,}\beta_1,\ldots,\beta_K)x).$$

By using the unique solution principle in the functional theory we obtain the following equality:

$$\lim_{x\to\infty}\left[\frac{F(x,\alpha_1,\ldots,\alpha_N,\beta_1,\ldots,\beta_K)}{F_{as}(x,\alpha_1,\ldots,\alpha_N,\beta_1,\ldots,\beta_K)}\right] = 1 \tag{4}$$

fter simple modification the equality (4) takes the form

$$\lim_{x\to\infty}\left(\frac{F(x+1,\alpha_1,\ldots,\alpha_N,\beta_1,\ldots,\beta_K)}{F(x,\alpha_1,\ldots,\alpha_N,\beta_1,\ldots,\beta_K)}\right) = \left(\frac{F_{as}(x+1,\alpha_1,\ldots,\alpha_N,\beta_1,\ldots,\beta_K)}{F_{as}(x,\alpha_1,\ldots,\alpha_N,\beta_1,\ldots,\beta_K)}\right) = \frac{e^{Y(x+1)}}{e^{Yx}} = \exp(Y)$$

where $Y = Y(\alpha_1,\ldots,\alpha_N,\beta_1,\ldots,\beta_K)$. Now, from the above equation the formulae

$$Y = \lim_{x\to\infty}\left(\ln\left(\frac{F(x+1,\alpha_1,\ldots,\alpha_N,\beta_1,\ldots,\beta_K)}{F(x,\alpha_1,\ldots,\alpha_N,\beta_1,\ldots,\beta_K)}\right)\right) \quad (5)$$

directly follows. Now, we have the following definition:

$$Y = trans_{"IDprocess"}(x,\alpha_1,\ldots,\alpha_N,\beta_1,\ldots,\beta_K) = \lim_{x\to\infty}\left(\ln\left(\frac{F(x+1,\alpha_1,\ldots,\alpha_N,\beta_1,\ldots,\beta_K)}{F(x,\alpha_1,\ldots,\alpha_N,\beta_1,\ldots,\beta_K)}\right)\right).$$

The outline of some Special trans functions derivation is based on the fact that the STFT can be applied for arbitrary nonlinear forms in a straightforward manner ([1-11]).

2. THE NONLINEAR FUNCTIONAL EQUATION FOR SOME VOLTAGE DISTRIBUTION DIODES NETWORKS

The subject of the theoretical analysis presented here is the nonlinear circuit from class of the nonlinear circuits described by equation (1) and given in Fig.1. The base equation for this network (Kirchoof low) takes the form

$$-i_{D1} + i_{R_1} + i_{C1} = -i_{D_2} + i_{R_2} + i_{C2} \quad (6)$$

where

$$\begin{aligned} i_{D1} &= i_{s_1}\left(e^{-\frac{U_{D_1}}{V_T}} - 1\right) \\ i_{D2} &= i_{s_2}\left(e^{-\frac{U_{D_2}}{V_T}} - 1\right) \\ ;i_{C1} &= \frac{U_{D1}}{R}e^{-\frac{t}{RC}}; \quad i_{C2} = \frac{U - U_{D1}}{R}e^{-\frac{t}{RC}} \end{aligned} \quad (7)$$

$i_{D1} = \frac{U_{D_1}}{R_1}$; $i_{D2} = \frac{U_{D_2}}{R_2}$; $U = U_{D_1} + U_{D_2}$ i_{s1}, i_{s2} are the saturation currents; V_T is the thermal voltage defined as $V_T = kT/e$, where k is the Boltzmann's constant; T is the junction absolute temperature in the degrees Kelvin; e is the electron charge. After adequate equations (6) and (7) substitutions we have

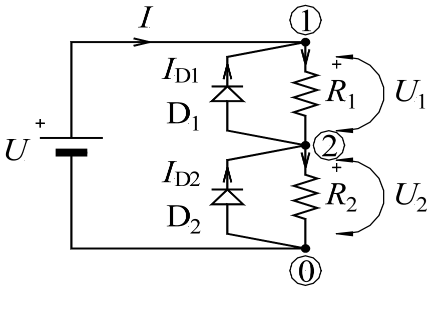

Fig.1. *Voltage distribution networks for diodes operating in series*

$$i_{s1}\left(1-e^{-\frac{U_{D_1}}{V_T}}\right) + \frac{U_{D_1}}{R_1} + \frac{U_{D1}}{R}e^{-\frac{t}{RC}} = i_{s2}\left(1-e^{-\frac{(U-U_{D_1})}{V_T}}\right) + \frac{U-U_{D_1}}{R_2} + \frac{U-U_{D1}}{R}e^{-\frac{t}{RC}} \quad (8)$$

After some modification equation (8) takes the form

$$Z\alpha e^{-Z} = e^{-2Z} + \beta e^{-Z} - \delta \quad (9)$$

where

$$\alpha = \frac{V_T}{R_1 i_{s1}}\left(1 + \frac{R_1}{R_2} + \frac{2R_1}{R}e^{-\frac{t}{RC}}\right); \quad Z = \frac{U_{D_1}}{V_T}$$
$$\beta = \frac{U}{R_2 i_{s1}} + \frac{i_{s2}}{i_{s1}} - 1 + \frac{U}{Ri_{s1}}e^{-\frac{t}{RC}}; \quad \delta = \frac{i_{s2}}{i_{s1}}e^{-\frac{U}{V_T}} \quad (10)$$

now, it is not difficulties to see that equation (9) takes the form

$$e^{-2Y} - aYe^{-Y} - b = 0 \quad (11)$$

where

$$a = \alpha e^{\frac{\beta}{\alpha}}; \quad b = \delta e^{2\frac{\beta}{\alpha}}; \quad Y = Z - \frac{\beta}{\alpha} \quad (12)$$

Let us note that circuits (Fig.2.) with capacity parallel with resistor are described by nonlinear functional equation (11) as well.

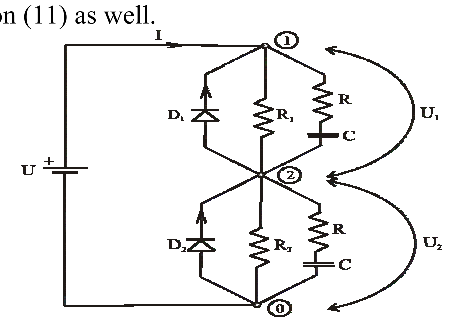

Fig.2. *Voltage distribution RC networks for diodes operating in series*

Namely, for this example we have the following equation

$$e^{-2\psi} - a_t\psi e^{-\psi} - b_t = 0$$

where

$$a_t = \alpha_t e^{\frac{\beta_t}{\alpha_t}}; \quad b_t = \delta e^{2\frac{\beta_t}{\alpha_t}}; \quad \psi = \theta - \frac{\beta_t}{\alpha_t}$$

$$\alpha_t = \frac{V_T}{R_1 i_{s1}}\left(1 + \frac{R_1}{R_2} + \frac{2R_1}{R}e^{-\frac{t}{RC}}\right)$$

$$\beta_t = \frac{U}{R_2 i_{s1}} + \frac{i_{s2}}{i_{s1}} - 1 + \frac{U}{Ri_{s1}}e^{-\frac{t}{RC}}$$

$$\delta = \frac{i_{s2}}{i_{s1}}e^{-\frac{U}{V_T}}; \quad \theta = \frac{U_{D1}}{V_T}$$

3. THE ANALYTICAL SOLUTION TO THE NONLINEAR FUNCTIONAL EQUATION (11)

In this section we attempt to find the analytical closed form solution of nonlinear functional equation (11).

Theorem **1.** If $a \in R^{+}$, $b \in R^{+}$ and b<1 the transcendental equation (11) has the analytical closed form solution

$$Y = trans_{VD}(a,b) \qquad (13)$$

where $trans_{VD}(a,b)$ is a new special tran function defined as

$$trans_{VD}(a,b) = \lim_{x\to\infty}\left[\ln\left(\frac{\varphi_{VD}(x+1,a,b)}{\varphi_{VD}(x,a,b)}\right)\right] \qquad (14)$$

where the function $\varphi_{VD}(x,a,b)$ takes the form

$$\varphi_{VD}(x,a,b) = \sum_{n=0}^{\left[\frac{x-1}{2}\right]}\sum_{k=0}^{n}\sum_{m=0}^{k}\sum_{p=0}^{n-k}\frac{(-1)^{k+m+p}\, n!\, b^{p}(x-m-k-1)^{m+1+p}}{(k-m)!\,m!\,(n-k-p)!\,p!\,(m+1+p)!\,a^{m+p}} \qquad (15)$$

where [(x-1)/2] denotes the greatest integer less or equal to (x-1)/2.

Proof: The nonlinear functional equation (11) can be identified with partial differential equation of type

$$\varphi_{VD}(x-2,a,b) - a\frac{\partial\varphi_{VD}(x-1,a,b)}{\partial x} - b\varphi_{VD}(x,a,b) = 0 \qquad (16)$$

where $\varphi_{VD}(x,a,b)$ is an arbitrary real function for x>0 and $\varphi_{VD}(x,a,b) = 0$ for x<0.

This partial differential equation for identification (EQID) is analytically solvable using a Laplace Transform. Thus, after a Laplace Transform the equation (16) takes the following form:

$$\Phi(s,a,b)e^{-2s} - as\Phi(s,a,b)e^{-s} - b\Phi(s,a,b) = -a\varphi_{o}e^{-s} \qquad (17)$$

where $\Phi(s,a,b) = L\{\varphi_{VD}(x,a,b)\}$. By using the elementary modification of equation (17) it is also easily to verified that

$$\Phi(s,a,b) = -\frac{a\varphi_{o}e^{-s}}{e^{-2s} - ase^{-s} - b} = \frac{\varphi_{o}e^{-s}}{s\left(1-\left(\frac{e^{-2s}}{as} - \frac{b}{as} - e^{-s} + 1\right)\right)}. \qquad (18)$$

Using the sum infinite series method, Laplace Transform (18) takes the form

$$\Phi(s,a,b) = \frac{\varphi_{o}e^{-s}}{s}\sum_{n=0}^{\infty}\left(1+\frac{e^{-2s}}{as} - e^{-s} - \frac{b}{as}\right)^{n}.$$

Now, using the well known binomial theorem approach the series expansion becomes

$$\Phi(s,a,b) = \frac{\varphi_{o}e^{-s}}{s}\sum_{n=0}^{\infty}\sum_{k=0}^{n}\frac{n!}{(n-k)!\,k!}\left(1-\frac{b}{as}\right)^{n-k}(-1)^{k}\left(1-\frac{e^{-s}}{as}\right)^{k}e^{-ks} =$$

$$\varphi_{o}\sum_{n=0}^{\infty}\sum_{k=0}^{n}\sum_{m=0}^{k}\sum_{p=0}^{n-k}\left[\frac{n!}{(k-m)!\,m!}\cdot\frac{(-1)^{k+m+p}e^{-(k+m+1)s}b^{p}}{(n-k-p)!\,p!\,a^{m+p}s^{m+1+p}}\right]. \qquad (19)$$

Then, we can invert term by term to obtain in the original domain x

$$\varphi_{VD}(x,a,b) = \sum_{n=0}^{\infty}\sum_{k=0}^{n}\sum_{m=0}^{k}\sum_{p=0}^{n-k}\frac{(-1)^{k+m+p}\, n!\, b^{p}(x-m-k-1)^{m+1+p}\, h(x-m-k-1)}{(k-m)!\,m!\,(n-k-p)!\,p!\,(m+1+p)!\,a^{m+p}}$$

where h(x-m-k-1) is the Heaviside's unit function.

Finally, applying the Laplace Transform, the analytical solution for the partial differential equation (16) can be written in the closed form representation

$$\varphi_{VD}(x,a,b) = \sum_{n=0}^{\left[\frac{x-1}{2}\right]}\sum_{k=0}^{n}\sum_{m=0}^{k}\sum_{p=0}^{n-k}\frac{(-1)^{k+m+p}\, n!\, b^{p}(x-m-k-1)^{m+1+p}}{(k-m)!\,m!\,(n-k-p)!\,p!\,(m+1+p)!\,a^{m+p}}. \qquad (20)$$

4. THE UNIQUE SOLUTION PRINCIPLE

Equation (20) is analytical closed-form solution to the partial differential equation (16). On the other hand, differential equation (16) has an asymptotic solution of the form

$$\varphi_{VD\,A}(x,a,b) = \varphi_{o}\exp(Yx). \qquad (21)$$

Applying the STFT ([1]-[11]), we obtain the following model of equalization in the form

$$\lim_{x\to\infty}\left(\frac{\varphi_{VD}(x,a,b)}{\varphi_{VDA}(x,a,b)}\right) = 1 \qquad (22)$$

and

$$\lim_{x\to\infty}\left(\frac{\varphi_{VD}(x+1,a,b)}{\varphi_{VD}(x,a,b)}\right) = \left(\frac{\varphi_{VDA}(x+1,a,b)}{\varphi_{VDA}(x,a,b)}\right) = \exp(Y) \Rightarrow Y = \lim_{x\to\infty}\left[\ln\left(\frac{\varphi_{VD}(x+1,a,b)}{\varphi_{VD}(x,a,b)}\right)\right] \qquad (23)$$

Consequently, we have $Y = trans_{VD}(a,b)$ where $trans_{VD}(a,b)$ a new special function is defined in (19). For practical calculation we have

$$\langle Y\rangle_{[G]} = \ln\left(\frac{\varphi_{VD}(x+1,a,b)}{\varphi_{VD}(x,a,b)}\right), \quad for \quad x > x_{o} \qquad (24)$$

or, more explicitly

$$\langle Y\rangle_{|G|} =$$
$$\langle trans_{VD}(a,b)\rangle_{|G|} =$$
$$\left\langle \ln\left(\frac{\sum_{n=0}^{\left[\frac{x}{2}\right]}\sum_{k=0}^{n}\sum_{m=0}^{k}\sum_{p=0}^{n-k} \frac{(-1)^{k+m+p} n! b^p (x-m-k)^{m+1+p}}{(k-m)! m! (n-k-p)! p! (m+1+p)! a^{m+p}}}{\sum_{n=0}^{\left[\frac{x-1}{2}\right]}\sum_{k=0}^{n}\sum_{m=0}^{k}\sum_{p=0}^{n-k} \frac{(-1)^{k+m+p} n! b^p (x-m-k-1)^{m+1+p}}{(k-m)! m! (n-k-p)! p! (m+1+p)! a^{m+p}}} \right) \right\rangle_{|G|} \tag{25}$$

where, G is the error function defined as: $G = e^{-2Y} - aYe^{-Y} - b$, x_o is value of x, when the error function G satisfies the inequality $|G| \le g_o \quad for \quad x \ge x_o$ where g_o is an arbitrary small positive real value. $\langle Y\rangle_{[G]}$ is value of Y given with [G] accurate digits ([1], [3]).

5. THE FINAL FORM OF THE ANALYTICAL SIGNAL ANALYSIS

Applying the STFT for nonlinear circuit, from equations (10), (12) and (23) we find the following expression

$$\langle U_{D1}\rangle_{|G|} = \left\langle V_T\left(trans_{VD}(a,b) + \frac{\beta}{\alpha}\right)\right\rangle_{|G|} \tag{26}$$
$$\langle U_{D2}\rangle_{|G|} = U - \left\langle V_T\left(trans_{VD}(a,b) + \frac{\beta}{\alpha}\right)\right\rangle_{|G|}$$

$$\langle i_{D1}\rangle_{|G|} = \left\langle i_{s1}\left(e^{-trans_{VD}(a,b) - \frac{\beta}{\alpha}} - 1\right)\right\rangle_{|G|} \tag{27}$$
$$\langle i_{D2}\rangle_{|G|} = \left\langle i_{s2}\left(e^{-\frac{U}{V_T} + trans_{VD}(a,b) + \frac{\beta}{\alpha}} - 1\right)\right\rangle_{|G|}$$

where $\langle trans_{VD}(a,b)\rangle_{|G|}$ is defined in equation (25).

6. RESULTS

To verify the theory a numerical example was carried out using the Mathematica program for equation (25). The obtained numerical results show a good agreement between analytical closed form solution calculation of equation (25), by Mathematica program and numerical results obtained by solver application [1].

7. CONCLUSIONS

A novel analytical approach has been proposed and signals for nonlinear circuit in Fig.1. are obtained exactly in analytical closed form ((26), (27)). An attractive feature of these formulae is possibility to obtain the gradients of the form:

$$\frac{\partial S_k}{\partial v_k}; \quad S_k = \{U_{D1}, U_{D2}, i_{D1}, i_{D2}\} \tag{28}$$
$$v_k = \{i_{s1}, i_{s2}, R_1, R_2, U, T\}$$

Namely, if we abstract that Special Trans Function genesis is complicated, we have the transcendental equation solutions obtainable in the simple forms ((13), (14) and (25)). For available data base of trans functions it is possible to obtain directly solutions. Consequently, this is an easy approach to the nonlinearity. On the other hand, from equation (25) we have possibility to obtain, for example, parameters described in (28). The proposed analytical parameters analysis for solving nonlinear problems gives advantages in the comparison with conventional methods - more easy and effective way of analytical nonlinear problems analysis, better clarity and with high accuracy ([1], [3], [6], and [11]). It is clear that this analytical parameter's analysis is impossible in the numerical approach to the nonlinear problem analysis. STFT analytical parameter's analysis could be applied for solving many different classes of nonlinear problems in applied physics and engineering domain, as it was done and verified in the Special Trans Function Theory (S.M. Perovich) application ([1]-[11] et al).

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