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# A THEOREM OF RECIPROCITY FOR THE ELECTRO-KINETIC STATE TAKING INTO ACCOUNT THE MAGNETIC FIELD

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**Abstract:** *The present paper establishes a reciprocity theorem for the electro-kinetic field inside a semiconductor medium placed in a uniform magnetic field. Having as basis this theorem, we can prove that the own resistance of a semiconductor environment does not depend on the electro-kinetic state in the presence of an outside magnetic field.*

**Key Words:** ***Electro-kinetic state, semiconductor enviroment, magnetic field***

## 1. OHM'S LAW

Due to the fact that in the presence of the magnetic field, the point form of Ohm's law gets new terms, the own resistance and the transfer resistance have some properties presented in [1,2]. The theorems of reciprocity for the electric and magnetic field allow defining the circuit parameters (R, L, C) in the most general conditions. The reciprocity of the electric field in the presence of a magnetic field has some features [3], also satisfying the Onsager-Casimir relation. A theorem of reciprocity has been demonstrated in [4], considering a particular case of a Corbino disk.

In this paper we have generalized a theorem of reciprocity of electro-kinetic field inside a semiconductor environment placed in a uniform magnetic field. On the basis of the theorem, it is possible to define the electric resistance of the semiconductor environment.

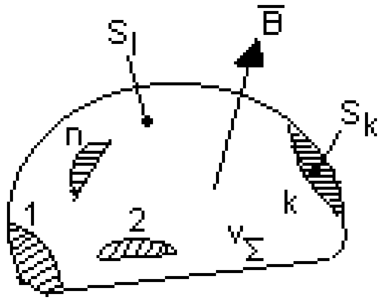


Fig.1

For a linear, homogeneous and isotropic semiconductor with an arbitrary geometry, fig.1, having the same temperature and placed in a uniform magnetic field, the point form of the Ohm's law is, [2]:

$$\mathbf{J} = K_1\mathbf{E} + K_2(\mathbf{E}\times\mathbf{B}) + K_3(\mathbf{E}\cdot\mathbf{B})\mathbf{B}\,, \qquad (1)$$

where $\mathbf{J}$ stands for the current density, $\mathbf{E}$ the electric field intensity, $\mathbf{B}$ the magnetic flux density and $K_1$, $K_2$, $K_3$ three coefficients that depend on the semiconductor material, on the temperature and on the value of the magnetic flux density. Considering a new electrokinetic state different from (1), where $\mathbf{J'}\neq\mathbf{J}$, $\mathbf{E'}\neq\mathbf{E}$, but having the same temperature and magnetic flux density, the point form of the Ohm's law becomes:

$$\mathbf{J'} = K_1\mathbf{E'} + K_2(\mathbf{E'}\times\mathbf{B}) + K_3(\mathbf{E'}\cdot\mathbf{B})\mathbf{B}\,. \qquad (2)$$

Multiplying (1) and (2) by $\mathbf{E'}$ respectively $\mathbf{E}$ and them substracting the two resulting relation, we obtain:

$$\mathbf{J}\cdot\mathbf{E'} - \mathbf{J'}\cdot\mathbf{E} = 2K_2(\mathbf{E'}\times\mathbf{E})\cdot\mathbf{B}\,. \qquad (3)$$

For a stationary state:

$div\,\mathbf{J} = div\,\mathbf{J'} = 0$, $curl\,\mathbf{E} = curl\,\mathbf{E'} = 0$, $\mathbf{E} = -grad\,V$, $\mathbf{E'} = -grad\,V'$, where V and V' represents the electric potential coresponding to the two electro-kinetic states.

Neglecting the magnetic field, generated by the conducting currents established in the semiconductor environment, towards the external magnetic field, we have $rot\,\mathbf{B} = 0$, respectively

$$\mathbf{B} = grad\,\varphi = \nabla\varphi\,, \qquad (4)$$

with φ representing a scalar function.

Using the identity:

$$\nabla\cdot(\varphi\,\mathbf{E'}\times\mathbf{E}) = \varphi\nabla\cdot(\mathbf{E'}\times\mathbf{E}) + (\mathbf{E'}\times\mathbf{E})\cdot\nabla\varphi\,,$$

where $\varphi\nabla\cdot(\mathbf{E'}\times\mathbf{E}) = \varphi\,\mathbf{E}\cdot(\nabla\times\mathbf{E'}) - \varphi\,\mathbf{E'}\cdot(\nabla\times\mathbf{E}) = 0$,

the right-hand side of (3) becomes:

$$2K_2(\mathbf{E'}\times\mathbf{E})\cdot\mathbf{B} = 2K_2\nabla(\varphi\,\mathbf{E'}\times\mathbf{E})\,.$$

Integrating the (3) for the entire volume of the semiconductor environment, we have:

$$\int_{v_\Sigma}(\mathbf{J}\cdot\mathbf{E'} - \mathbf{J'}\cdot\mathbf{E})dv = 2K_2\int_{v_\Sigma}\nabla(\varphi\,\mathbf{E'}\times\mathbf{E})dv\,. \qquad (5)$$

## 2. THE THEOREM OF RECIPROCITY

Using the vector identities such as:

$$\nabla \cdot (V'\mathbf{J}) = V'\nabla \cdot \mathbf{J} + \mathbf{J} \cdot \nabla V' = -\mathbf{J} \cdot \mathbf{E}', \qquad (6)$$

we obtain:

$$\int_{v_\Sigma} (\mathbf{J} \cdot \mathbf{E}' - \mathbf{J}' \cdot \mathbf{E})\, dv = \int_\Sigma (V\mathbf{J}' - V'\mathbf{J}) \cdot \mathbf{dS}, \qquad (7)$$

where $v_\Sigma$ is the volume enclosed by the closed surface Σ consisting of the electrodes surface $S_k$ and the free surface $S_l$ uncovered by electrodes.

The surface $S_l$ represents a field surface for the current density **J**, where $\mathbf{J} \cdot \mathbf{dS} = \mathbf{J}' \cdot \mathbf{dS} = 0$, and assuming the electrodes conductivity being much greater than the semiconductor conductivity, the $S_k$ surfaces become an equipotential surfaces. From (7), we have:

$$\int_\Sigma (V\mathbf{J}' - V'\mathbf{J}) \cdot \mathbf{dS} = \sum_{k=1}^{n} \left( V_k I_k' - V_k' I_k \right), \qquad (8)$$

where $I_k' = \int_{S_k} \mathbf{J}' \cdot \mathbf{dS}$ and $I_k = \int_{S_k} \mathbf{J} \cdot \mathbf{dS}$ represent the outward-flowing current from the semiconductor environment through the electrodes surface k.

Using the divergence theorem, the right-hand side of (5) becomes:

$$\int_{v_\Sigma} \nabla (\varphi\, \mathbf{E}' \times \mathbf{E})\, dv = \int_\Sigma \varphi (\mathbf{E}' \times \mathbf{E}) \cdot \mathbf{dS}. \qquad (9)$$

The electrode surfaces being considered as equipotential surfaces, the electric field intensity is orthogonal on the electrode surfaces $S_k$, $\mathbf{E} = E\mathbf{n}$, $\mathbf{E}' = E'\mathbf{n}$, and $\mathbf{E}' \times \mathbf{E} = 0$.

If the two different electro-kinetic states are established between the same electrodes but with different values of the current, then we can consider on the free $S_l$ surface that the identity $\mathbf{J}' = k\mathbf{J}$ is satisfied, where k is a constant of proportionality. Hence, for the $S_l$ surfaces, the relation (2) becomes:

$$\mathbf{J}' = k\mathbf{J} = K_1\mathbf{E}' + K_2(\mathbf{E}' \times \mathbf{B}) + K_3(\mathbf{E}' \cdot \mathbf{B})\mathbf{B},$$

which, comparing with (1) after multiplying by k, leads to identities $\mathbf{E}' = k\mathbf{E}$ and $\mathbf{E}' \times \mathbf{E} = 0$ also for the free surfaces $S_l$. As a consequence, the right-hand side of (9) and implicitly of (5) is zero, and from (5), (7) and (8), there results the theorem of reciprocity:

$$\sum_{k=1}^{n} V_k I_k' = \sum_{k=1}^{n} V_k' I_k. \qquad (10)$$

On the basis of the reciprocity theorem (10), it is possible to define the own resistance compared to the electrodes 1 and 2 in a semiconductor environment of arbitrary geometry, in a uniform magnetic field.

## 3. CONCLUSIONS

The semiconductor environments of arbitrary geometry where the electro-kinetic field is no longer plane-parallel, placed in uniform magnetic fields having the same temperature, we can prove a reciprocity theorem only if the two electro-kinetic states established according to the same supplying electrodes. Having as basis this reciprocity theorem we can define the resistance of the system.

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