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NEW ALGORITHM FOR RECONSTRUCTION OF BAND-LIMITED AC SIGNALS FROM IRREGULARY SPACED SETS OF INTEGRATED VALUES

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Abstract: *The problem of AC signal reconstruction is addressed in this paper. This study propose a practical and new algorithm for signal reconstruction, and discuss potential applications to recover band-limited AC signals (with known frequency spectrum but unknown amplitudes and phases) from irregularly spaced sets of integrated values of processed signals. Based on the value of the integer of the original input (analogue) signal, a reconstruction of its basic parameters is done by the means of derived analytical and summarized expressions. Computer simulation demonstrating the effectiveness of these algorithms, are also presented.*

Key Words: ***signal reconstruction, AC signals, integrated values, computer simulation.***

1. INTRODUCTION

Reconstructing a signal from measurements is a central problem in many signal processing applications. Often due to physical limitations, the available data is insufficient to achieve high resolution reconstruction. This is especially important for the problems that occur during the conversion of the observed AC signals, due to the non-ideal nature of the AD converter that is used here, as well as the transducer and the other devices used in the process.

In signal processing, reconstruction usually means the determination of an original continuous signal from a sequence of equally spaced samples [1]. Any real signal which is transmitted along some form of channel which will have a finite bandwidth. As a result the received signal's spectrum cannot contain any frequencies above some maximum value, f_{max}. As a consequence we can specify everything we know about the signal spectrum in terms of a d.c. level plus the amplitudes and phases of just M frequencies — i.e. all the information we have about the spectrum can be specified by just $2M+1$ numbers.

On the other hand sampling consists of a „sample and hold"(or „track and hold") circuit followed by conversion to digital code word, and is implemented using an ADC (analogue to digital converter). A sample and hold circuit is a possible source of systematic errors. In [2], attention is paid to the possibility of the elimination of a separate circuit for sample and hold, which in itself is an original approach. This paper provided the platform for the development of an entirely new algorithm for processing slow-changing AC signals. This is explained in the basic equations (3)-(7) [2]. The algorithm in which the reconstruction of the signal is done, through a system of linear equations and conditions (9) is explained in detail in [3].

In this paper we propose an efficient implementation of the algorithm for AC signal reconstruction that yields a significant improvement in computational efficiency over the standard implementation. The approach is based on the use of the values that were obtained as a result of the integer-processing of the continual input signal, in precisely defined time periods; this kind of integer-processing was done as many times as was needed to enable the reconstruction of the complex signal that is the subject of the processing operation.

2. PROPOSED ALGORITHM

Let us assume that the input signal of the fundamental frequency f is band limited to the first M harmonic components (equation (3) [4]). This form of continuous voltage signal with a complex harmonic content can be represented as a sum of the Fourier components as follows:

$$v_{input}(t) = V_I + \sum_{k=1}^{M} V_k \sin(k2\pi ft + \psi_k) \qquad (1)$$

Here V_I is the average value of the input voltage; V_k is the amplitude voltage of the kth harmonic; k is the number of the harmonic; and ψ_k is the phase angle of the kth harmonic of the voltage.

Applying equations (7) [2], and by forming a system of equations of the same form, in order to determine the $2M+1$ unknowns (amplitudes and phases of the M harmonic, as well as the average value of the signal):

$$A_0V_I+\sum_{k=1}^{n}A_kV_k\left(\sin\alpha_k\cos\psi_k+\cos\alpha_k\sin\psi_k\right)=B^{(k)}$$

$$\frac{1}{k}\sin\left(k\pi f\,2^n t_c\right)=A_k,\quad (k=1,2,\ldots,M) \tag{2}$$

$$k\pi f\left(2^{n+1}t_c+2t_0\right)=\alpha_k;\quad \pi f\,2^n t_c=A_0;\quad \pi f V_{REF}\left(t_{2k}-t_{1k}\right)=B^{(k)}$$

where $T_2 = t_{2k} - t_{1k} = it_c$, the time in which the counter registered i clock impulses, V_{REF} is reference voltage, $B^{(k)}$ is result of input signal integration, t_c is the sampling rate with which the counter inside the AD convertor does the counting, 2^n is the total number of the state that can be occupied by this counter (n-bit), while t_0 is the initial moment of the conversion process. The system of $2M+1$ (unknown parameters) equations we can represent as:

$$A_0V_I+\sum_{k=1}^{M}A_kV_k\left(\sin\alpha_k^{(1)}\cos\psi_k+\cos\alpha_k^{(1)}\sin\psi_k\right)=B^{(1)}$$

$$A_0V_I+\sum_{k=1}^{M}A_kV_k\left(\sin\alpha_k^{(2)}\cos\psi_k+\cos\alpha_k^{(2)}\sin\psi_k\right)=B^{(2)} \tag{3}$$

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$$A_0V_I+\sum_{k=1}^{M}A_kV_k\left(\sin\alpha_k^{(2M+1)}\cos\psi_k+\cos\alpha_k^{(2M+1)}\sin\psi_k\right)=B^{(2M+1)}$$

The system matrix is:

$$E_{2M+1}^{system}=A_0\left(A_1\ldots A_M\right)^2E_{2M+1} \tag{4}$$

where is:

$$E_{2M+1}=\begin{vmatrix}1 & \sin\alpha_1^{(1)} & \sin\alpha_2^{(1)} & \ldots & \sin\alpha_M^{(1)} & \cos\alpha_1^{(1)} & \cos\alpha_2^{(1)} & \ldots & \cos\alpha_M^{(1)}\\ 1 & \sin\alpha_1^{(2)} & \sin\alpha_2^{(2)} & \ldots & \sin\alpha_M^{(2)} & \cos\alpha_1^{(2)} & \cos\alpha_2^{(2)} & \ldots & \cos\alpha_M^{(2)}\\ . & . & . & & & & & & \\ 1 & \sin\alpha_1^{(2M+1)} & \sin\alpha_2^{(2M+1)} & \ldots & \sin\alpha_M^{(2M+1)} & \cos\alpha_1^{(2M+1)} & \cos\alpha_2^{(2M+1)} & \ldots & \cos\alpha_M^{(2M+1)}\end{vmatrix}=$$
$$=\begin{vmatrix}1 & \sin\varphi_1 & \sin 2\varphi_1 & \ldots & \sin M\varphi_1 & \cos\varphi_1 & \cos 2\varphi_1 & \ldots & \cos M\varphi_1\\ 1 & \sin\varphi_2 & \sin 2\varphi_2 & \ldots & \sin M\varphi_2 & \cos\varphi_2 & \cos 2\varphi_2 & \ldots & \cos M\varphi_2\\ . & . & . & & & & & & \\ 1 & \sin\varphi_{2M+1} & \sin 2\varphi_{2M+1} & \ldots & \sin M\varphi_{2M+1} & \cos\varphi_{2M+1} & \cos 2\varphi_{2M+1} & \ldots & \cos M\varphi_{2M+1}\end{vmatrix} \tag{5}$$

Here is:

$$\varphi_i=\frac{i\pi f_0\left(2^{n+1}t_c+2t_0\right)}{2M+1};\, i\in\{1,2,\ldots,2M+1\} \tag{6}$$

The A_k and φ_i are the variables which are the result of integration in real dual-slope ADC of the signal defined with equation (2) [4].

Starting from the well-known Van der Monde [5] determinant:

$$\Delta_M=w(x_1,x_2,\ldots,x_M)=\begin{vmatrix}1 & x_1 & x_1^2 & \ldots & x_1^{M-1}\\ 1 & x_2 & x_2^2 & \ldots & x_2^{M-1}\\ . & . & . & & \\ 1 & x_M & x_M^2 & \ldots & x_M^{M-1}\end{vmatrix}=\prod_{j=k+1}^{M}\prod_{k=1}^{M-1}\left(x_j-x_k\right)=S_{1,M} \tag{7}$$

If we introduce the following symbol for the co-determinant:

$$\Delta_{11}^{(M)}=\begin{vmatrix}x_2 & x_2^2 & \ldots & x_2^{M-1}\\ . & . & . & \\ x_M & x_M^2 & \ldots & x_M^{M-1}\end{vmatrix}=\left(x_2x_3\ldots x_M\right)\begin{vmatrix}1 & x_2 & \ldots & x_2^{M-2}\\ . & . & . & \\ 1 & x_M & \ldots & x_M^{M-2}\end{vmatrix}=\left(x_2x_3\ldots x_M\right)S_{2,M}= \tag{8}$$
$$=\left(x_2x_3\ldots x_M\right)\prod_{j=k+1}^{M}\prod_{k=2}^{M-1}\left(x_j-x_k\right)$$

In a similar way, it can be concluded that:

$$-\Delta_{12}^{(M)}=\begin{vmatrix}1 & x_2^2 & \ldots & x_2^{M-1}\\ 1 & x_3^2 & \ldots & x_3^{M-1}\\ . & . & . & \\ 1 & x_{M-1}^2 & \ldots & x_{M-1}^{M-1}\\ 1 & x_M^2 & \ldots & x_M^{M-1}\end{vmatrix}=\left(x_M-x_2\right)\left(x_M-x_3\right)\ldots\left(x_M-x_{M-1}\right)\left(x_M-\alpha_{12}^{(M)}\right)\Delta_{12}^{(M-1)} \tag{9}$$

where α coefficient is still unknown, and will be determined through the iterative procedure, while:

$$(-1)^{M-1}\left(x_2x_3\ldots x_{M-1}\right)\alpha_{12}^{(M)}\Delta_{12}^{(M)}=(-1)^M\begin{vmatrix}x_2^2 & \ldots & x_2^{M-1}\\ x_3^2 & \ldots & x_3^{M-1}\\ . & . & .\\ x_{M-1}^2 & \ldots & x_{M-1}^{M-1}\end{vmatrix}=(-1)^M\left(x_2x_3\ldots x_{M-1}\right)^2w\left(x_2,\ldots,x_{M-1}\right) \tag{10}$$

This means that the following written expression can be formulated:

$$\alpha_{12}^{(M)}\Delta_{12}^{(M-1)}=-\left(x_2\ldots x_{M-1}\right)S_{2,M-1} \tag{11}$$

From this, it follows that:

$$\Delta_{12}^{(M-1)}=\left(x_{M-1}-x_2\right)\left(x_{M-1}-x_3\right)\ldots\left(x_{M-1}-x_{M-2}\right)\left(x_{M-1}-\alpha_{12}^{(M-1)}\right)\Delta_{12}^{(M-2)} \tag{12}$$

$$(-1)^{M-2}\left(x_2\ldots x_{M-2}\right)\alpha_{12}^{(M-1)}\Delta_{12}^{(M-2)}=(-1)^{M-1}\left(x_2\ldots x_{M-2}\right)^2S_{2,M-2} \tag{13}$$

$$\alpha_{12}^{(M-1)}\Delta_{12}^{(M-2)}=-\left(x_2\ldots x_{M-2}\right)S_{2,M-2} \tag{14}$$

By continuing the same procedure (by reducing the order of the determinant), we get the following:

$$\Delta_{12}^{(4)}=\left(x_4-x_3\right)\left(x_4-x_2\right)\left(x_4-\alpha_{12}^{(4)}\right)\left(x_3-x_2\right)\left(x_3+x_2\right) \tag{15}$$

$$-x_2x_3\alpha_{12}^{(4)}\left(x_3-x_2\right)\left(x_3+x_2\right)=\begin{vmatrix}x_2^2 & x_2^3\\ x_3^2 & x_3^3\end{vmatrix}=\left(x_2x_3\right)^2\left(x_3-x_2\right)\Rightarrow\alpha_{12}^{(4)}=-\frac{x_2x_3}{x_2+x_3} \tag{16}$$

where:

$$\Delta_{12}^{(4)}=S_{2,4}T_{2,4};\quad T_{2,4}=x_4x_2+x_4x_3+x_3x_2 \tag{17}$$

Now, if we go back, to determine the unknown coefficients, we will get the following:

$$\Delta_{12}^{(5)}=\left(x_5-x_2\right)\left(x_5-x_3\right)\left(x_5-x_4\right)\left(x_5-\alpha_{12}^{(5)}\right)\Delta_{12}^{(4)} \tag{18}$$

$$\alpha_{12}^{(5)}\Delta_{12}^{(4)}=-\left(x_2x_3x_4\right)S_{2,4} \tag{19}$$

$$\Delta_{12}^{(5)}=S_{2,5}T_{3,5};\quad T_{3,5}=x_5x_4x_2+x_5x_4x_3+x_5x_3x_2+x_4x_3x_2 \tag{20}$$

After a sufficient number of steps, we get:

$$\Delta_{12}^{(M-1)}=S_{2,M-1}T_{M-3,M-1} \tag{21}$$

$$\alpha_{12}^{(M)}=-\frac{x_2x_3\ldots x_{M-1}}{T_{M-3,M-1}}\Rightarrow-\Delta_{12}^{(M)}=S_{2,M}T_{M-2,M} \tag{22}$$

Finally, we can write that:

$$\Delta_{12}^{(M)}=-S_{2,M}\left(x_2x_3\ldots x_M\right)\sum\frac{1}{\left(x_2x_3\ldots x_M\right)_1} \tag{23}$$

where:

$$\sum\frac{1}{\left(x_2x_3\ldots x_M\right)_1}=\sum_{j=2}^{M}\frac{1}{x_j}=\frac{1}{x_2}+\frac{1}{x_3}+\ldots+\frac{1}{x_M} \tag{24}$$

If we assume that:

$$\Delta_{13}^{(M)}=\begin{vmatrix}1 & x_2 & x_2^3 & \ldots & x_2^{M-1}\\ 1 & x_3 & x_3 & \ldots & x_3^{M-1}\\ . & . & . & & \\ 1 & x_{M-1} & x_{M-1}^3 & \ldots & x_{M-1}^{M-1}\\ 1 & x_M & x_M^3 & \ldots & x_M^{M-1}\end{vmatrix}= \tag{25}$$
$$=\left(x_M-x_2\right)\left(x_M-x_3\right)\ldots\left(x_M-x_{M-1}\right)\left(x_M-\alpha_{13}^{(M)}\right)\Delta_{13}^{(M-1)}$$

where:

$$(-1)^{M-1}\left(x_2\ldots x_{M-1}\right)\alpha_{13}^{(M)}\Delta_{13}^{(M-1)}=(-1)^M\left(x_2\ldots x_{M-1}\right)\begin{vmatrix}1 & x_2^2 & \ldots & x_2^{M-2}\\ 1 & x_3^2 & \ldots & x_3^{M-2}\\ . & . & . & \\ 1 & x_M^2 & \ldots & x_M^{M-2}\end{vmatrix} \tag{26}$$

$$\alpha_{13}^{(M)}\Delta_{13}^{(M-1)}=-\begin{vmatrix}1 & x_2^2 & \ldots & x_2^{M-2}\\ 1 & x_3^2 & \ldots & x_3^{M-2}\\ . & . & . & \\ 1 & x_{M-1}^2 & \ldots & x_{M-1}^{M-2}\end{vmatrix}=-S_{2.M-1}\left(x_2x_3\ldots x_{M-1}\right)\sum\frac{1}{\left(x_2x_3\ldots x_{M-1}\right)_1} \tag{27}$$

$$\Delta_{13}^{(M-1)}=\left(x_{M-1}-x_2\right)\left(x_{M-1}-x_3\right)\ldots\left(x_{M-1}-x_{M-2}\right)\left(x_{M-1}-\alpha_{13}^{(M-1)}\right)\Delta_{13}^{(M-2)} \tag{28}$$

$$\alpha_{13}^{(M-1)}\Delta_{13}^{(M-2)}=-S_{2,M-2}\left(x_2\ldots x_{M-2}\right)\sum\frac{1}{\left(x_2x_3\ldots x_{M-2}\right)_1} \tag{29}$$

$$\Delta_{13}^{(4)}=(x_4-x_2)(x_4-x_3)(x_4-\alpha_{13}^{(4)})\Delta_{13}^{(3)}=(x_4-x_2)(x_4-x_3)(x_3-x_2)(x_4-\alpha_{13}^{(4)}) \quad (30)$$

Apart from this, the following is applied:

$$-(x_2x_3)(x_3-x_2)\alpha_{13}^{(4)}=(x_2x_3)(x_3-x_2)(x_3+x_2)\Rightarrow \alpha_{13}^{(4)}=-(x_2x_3) \quad (31)$$

From now on, we can write that:

$$\Delta_{13}^{(4)}=S_{2,4}(x_2+x_3+x_4)=S_{2,4}T_{1,4} \quad (32)$$

$$\Delta_{13}^{(5)}=(x_5-x_2)(x_5-x_3)(x_5-x_4)(x_5-\alpha_{13}^{(5)})\Delta_{13}^{(4)} \quad (33)$$

$$\alpha_{13}^{(5)}\Delta_{13}^{(4)}=-S_{2,4}(x_2x_3x_4)\sum\frac{1}{(x_2x_3x_4)_1}=\alpha_{13}^{(5)}S_{2,4}T_{1,4}\Rightarrow \alpha_{13}^{(5)}=-\frac{x_4x_2+x_4x_3+x_3x_2}{x_2+x_3+x_4} \quad (34)$$

What we conclude from this is:

$$\Delta_{13}^{(5)}=S_{2,5}T_{2,5} \quad (35)$$

After a sufficient number of iterations, it can be concluded that:

$$\Delta_{13}^{(M-1)}=S_{2,M-1}T_{M-4,M-1} \quad (36)$$

$$\alpha_{13}^{(M)}=-\frac{x_2x_3\ldots x_{M-1}}{T_{M-4,M-1}}\sum\frac{1}{(x_2x_3\ldots x_{M-1})_1}\Rightarrow \Delta_{13}^{(M)}=S_{2,M}T_{M-3,M} \quad (37)$$

Finally, we get:

$$\Delta_{13}^{(M)}=S_{2,M}(x_2x_3\ldots x_M)\sum\frac{1}{(x_2x_3\ldots x_M)_2} \quad (38)$$

(the summing is done for all the two's).

For the next co-determinant in this succession, we can write that:

$$-\Delta_{14}^{(M)}=\begin{vmatrix}1 & x_2 & x_2^2 & x_2^4 & \ldots & x_2^{M-1}\\ 1 & x_3 & x_3^2 & x_3^4 & \ldots & x_3^{M-1}\\ . & . & . & & & \\ 1 & x_{M-1} & x_{M-1}^2 & x_{M-1}^4 & \ldots & x_{M-1}^{M-1}\\ 1 & x_M & x_M^2 & x_M^4 & \ldots & x_M^{M-1}\end{vmatrix}= \quad (39)$$

$$=(x_M-x_2)(x_M-x_3)\ldots(x_M-x_{M-1})(x_M-\alpha_{14}^{(M)})\Delta_{14}^{(M-1)}$$

$$(-1)^{M-1}(x_2\ldots x_{M-1})\alpha_{14}^{(M)}\Delta_{14}^{(M-1)}=$$

$$=(-1)^M(x_2\ldots x_{M-1})\begin{vmatrix}1 & x_2 & x_2^3 & \ldots & x_2^{M-2}\\ . & . & . & & \\ 1 & x_{M-1} & x_{M-1}^3 & \ldots & x_{M-1}^{M-2}\end{vmatrix}= \quad (40)$$

$$=(-1)^M(x_2\ldots x_{M-1})\Delta_{13}^{(M-1)}$$

From this, it can be concluded that:

$$\alpha_{14}^{(M)}\Delta_{14}^{(M-1)}=-S_{2,M-1}T_{M-4,M-1} \quad (41)$$

$$\Delta_{14}^{(M-1)}=(x_{M-1}-x_2)(x_{M-1}-x_3)\ldots(x_{M-1}-x_{M-2})(x_{M-1}-\alpha_{14}^{(M-1)})\Delta_{14}^{(M-2)} \quad (42)$$

$$\alpha_{14}^{(M-1)}\Delta_{14}^{(M-2)}=-S_{2,M-2}T_{M-5,M-2} \quad (43)$$

After conducting an iterative procedure, with which we reduce the order of the obtained expressions, we get the following:

$$\Delta_{14}^{(5)}=(x_5-x_2)(x_5-x_3)(x_5-\alpha_{14}^{(5)})(x_5-x_4)(x_4-x_2)(x_4-x_3)(x_3-x_2)$$

$$(x_4x_3x_2)\alpha_{14}^{(5)}(x_3-x_2)(x_4-x_3)(x_4-x_2)=-(x_4x_3x_2)\begin{vmatrix}1 & x_2 & x_2^3\\ 1 & x_3 & x_3^3\\ 1 & x_4 & x_4^3\end{vmatrix} \quad (44)$$

where:

$$\begin{vmatrix}1 & x_2 & x_2^3\\ 1 & x_3 & x_3^3\\ 1 & x_4 & x_4^3\end{vmatrix}=(x_4-x_2)(x_4-x_3)(x_3-x_2)(x_4-\beta) \quad (45)$$

What is also applied is that:

$$-(x_2x_3)\beta(x_3-x_2)=(x_2x_3)(x_3-x_2)(x_3+x_2)\Rightarrow$$
$$\beta=-(x_2+x_3);\ \alpha_{14}^{(5)}=-(x_2+x_3+x_4) \quad (46)$$

$$\Delta_{14}^{(5)}=S_{2,5}T_{1,5};\quad \Delta_{14}^{(6)}=S_{2,6}T_{2,6} \quad (47)$$

An analogous conclusion is that:

$$\Delta_{14}^{(M-1)}=S_{2,M-1}T_{M-5,M-1};\ \alpha_{14}^{(M)}=-\frac{T_{M-4,M-1}}{T_{M-5,M-1}};\ \Delta_{14}^{(M)}=-S_{2,M}T_{M-4,M} \quad (48)$$

If we perform a substitution here, we get:

$$\Delta_{14}^{(M)}=-S_{2,M}(x_2x_3\ldots x_M)\sum\frac{1}{(x_2x_3\ldots x_M)_3} \quad (49)$$

(the summing is done by all of the threes).

By repeating the procedure described above, we can come to the following equations:

$$\Delta_{1k}^{(M)}=(-1)^{k+1}S_{2,M}(x_2x_3\ldots x_M)\sum\frac{1}{(x_2x_3\ldots x_M)_{k-1}} \quad (50)$$

$$\Delta_{1M}^{(M)}=(-1)^{M+1}S_{2,M}(x_2x_3\ldots x_M)\sum\frac{1}{(x_2x_3\ldots x_M)_{M-1}} \quad (51)$$

It is absolutely clear that an analogous procedure can be conducted in order to find a solution to the determinants that have the form of $\Delta_{lk}^{(M)}$, where l takes the values from the set $2\le l\le 2M+1$.

In the case of the system of equations that has been formed following the suggested concept of processing the input AC signals, what we get is that, instead of the x variables in the above expressions, it is necessary to take in the trigonometric values for $\varphi_1,\varphi_2,\varphi_3,\ldots,\varphi_{2M+1}$. In that case, the first row of the coordinate formed in this way can be represented in the following form:

$$E_{2M+1}=\begin{vmatrix}1 & \sin\varphi_1 & \sin 2\varphi_1 & \ldots & \sin M\varphi_1 & \cos\varphi_1 & \cos 2\varphi_1 & \ldots & \cos M\varphi_1\end{vmatrix} \quad (52)$$

The given determinant (i.e. its first row) can be transformed in the following way (by using *Euler's* formulas):

$$\begin{aligned}
E_{2M+1}&=\frac{1}{(2i)^M}\frac{1}{2^M}\begin{vmatrix}1 & e^{\varphi_1 i}-e^{-\varphi_1 i} & e^{2\varphi_1 i}-e^{-2\varphi_1 i} & \ldots & e^{M\varphi_1 i}-e^{-M\varphi_1 i} & e^{\varphi_1 i}+e^{-\varphi_1 i} & e^{2\varphi_1 i}+e^{-2\varphi_1 i} & \ldots & e^{M\varphi_1 i}+e^{-M\varphi_1 i}\end{vmatrix}=\\
&=\frac{1}{(2i)^M}\begin{vmatrix}1 & e^{\varphi_1 i}-e^{-\varphi_1 i} & e^{2\varphi_1 i}-e^{-2\varphi_1 i} & \ldots & e^{M\varphi_1 i}-e^{-M\varphi_1 i} & e^{\varphi_1 i} & e^{2\varphi_1 i} & \ldots & e^{M\varphi_1 i}\end{vmatrix}=\\
&=\frac{(-1)^M}{2^M i^M}\begin{vmatrix}1 & e^{-\varphi_1 i} & e^{-2\varphi_1 i} & \ldots & e^{-M\varphi_1 i} & e^{\varphi_1 i} & e^{2\varphi_1 i} & \ldots & e^{M\varphi_1 i}\end{vmatrix}=\\
&=\frac{(-1)^M(-1)^M(-i)^M}{2^M}\begin{vmatrix}e^{-\varphi_1 i} & e^{-2\varphi_1 i} & \ldots & e^{-M\varphi_1 i} & 1 & e^{\varphi_1 i} & e^{2\varphi_1 i} & \ldots & e^{M\varphi_1 i}\end{vmatrix}=\\
&=\frac{(-i)^M}{2^M}(-1)^{\frac{(M-1)M}{2}}\begin{vmatrix}e^{-M\varphi_1 i} & e^{-(M-1)\varphi_1 i} & \ldots & e^{-2\varphi_1 i} & e^{-\varphi_1 i} & 1 & e^{\varphi_1 i} & e^{2\varphi_1 i} & \ldots & e^{M\varphi_1 i}\end{vmatrix}=\\
&=\frac{(-1)^{\frac{M(M+1)}{2}}i^M}{2^M}e^{-M(\varphi_1+\ldots+\varphi_{2M+1})i}\begin{vmatrix}1 & e^{\varphi_1 i} & e^{2\varphi_1 i} & \ldots & e^{(2M)\varphi_1 i}\end{vmatrix}=\\
&=\frac{(-1)^{\frac{M(M+1)}{2}}i^M}{2^M}\prod_{j=k+1}^{2M+1}\prod_{k=1}^{2M}\left(e^{\frac{\varphi_j+\varphi_k}{2}i}2i\sin\frac{\varphi_j-\varphi_k}{2}\right)e^{-M(\varphi_1+\ldots+\varphi_{2M+1})i}=\\
&=\frac{(-1)^{\frac{M(M+1)}{2}}i^M}{2^M}e^{-M(\varphi_1+\ldots+\varphi_{2M+1})i}e^{M(\varphi_1+\ldots+\varphi_{2M+1})i}2^{\frac{2M(2M+1)}{2}}i^{\frac{2M(2M+1)}{2}}\prod_{j=k+1}^{2M+1}\prod_{k=1}^{2M}\sin\frac{\varphi_j-\varphi_k}{2}\\
&=(-1)^{\frac{M(M+1)}{2}}2^{2M^2}\prod_{j=k+1}^{2M+1}\prod_{k=1}^{2M}\sin\frac{\varphi_j-\varphi_k}{2}
\end{aligned} \quad (53)$$

The co-determinants that are necessary to reach the solution of the given system of equations (3) are:

$$E_{2M+1,1}=\begin{vmatrix}B^{(1)} & \sin\varphi_1 & \sin 2\varphi_1 & \ldots & \sin M\varphi_1 & \cos\varphi_1 & \cos 2\varphi_1 & \ldots & \cos M\varphi_1\\ B^{(2)} & \sin\varphi_2 & \sin 2\varphi_2 & \ldots & \sin M\varphi_2 & \cos\varphi_2 & \cos 2\varphi_2 & \ldots & \cos M\varphi_2\\ . & . & . & & & & & & \\ B^{(2M+1)} & \sin\varphi_{2M+1} & \sin 2\varphi_{2M+1} & \ldots & \sin M\varphi_{2M+1} & \cos\varphi_{2M+1} & \cos 2\varphi_{2M+1} & \ldots & \cos M\varphi_{2M+1}\end{vmatrix} \quad (54)$$

and so on. The co-determinants given above, based on the following development can be write as:

$$E_{2M+1,1}=B^{(1)}E_{2M+1}^{(1)}+B^{(2)}E_{2M+1}^{(2)}+\ldots+B^{(2M+1)}E_{2M+1}^{(2M)} \quad (55)$$

By using the same procedure as with the previous determinants we obtain:

$$E_{2M+1}^{(1)}=(-1)^{\frac{(M-1)M}{2}}2^{2M(M-1)+1}\prod_{j=k+1}^{2M+1}\prod_{k=1}^{2M}\sin\frac{\varphi_j-\varphi_k}{2}\cdot$$
$$\cdot\sum_M\cos\left\{\frac{1}{2}(\varphi_2+\ldots+\varphi_{2M+1})-(\varphi_2+\ldots+\varphi_{2M+1})_M\right\} \quad (56)$$

(the summing is done by all of the *M's* of the set $\{\varphi_2, \varphi_3, ..., \varphi_{2M+1}\}$).

$$E_{2M+1}^{(2)} = (-1)^{\frac{(M-1)M}{2}} 2^{2M(M-1)+1} \prod_{j=k+1}^{2M+1}\prod_{k=2}^{2M} \sin\frac{\varphi_j - \varphi_k}{2} \cdot \sum\nolimits_{M-1} \sin\left\{\frac{1}{2}(\varphi_2 + ... + \varphi_{2M+1}) - (\varphi_2 + ... + \varphi_{2M+1})_{M-1}\right\} \quad (57)$$

$$E_{2M+1}^{(3)} = (-1)^{\frac{M(M-1)}{2}} 2^{2M(M-1)+1} \prod_{j=k+1}^{2M+1}\prod_{k=2}^{2M} \sin\frac{\varphi_j - \varphi_k}{2} \cdot \sum\nolimits_{M-2} \sin\left\{\frac{1}{2}(\varphi_2 + ... + \varphi_{2M+1}) - (\varphi_2 + ... + \varphi_{2M+1})_{M-2}\right\} \quad (58)$$

For every $3 \le p \le M$ we can write:

$$E_{2M+1}^{(p)} = (-1)^{\frac{M(M-1)}{2}} 2^{2M^2-2M+1} \prod_{j=k+1}^{2M+1}\prod_{k=2}^{2M} \sin\frac{\varphi_j - \varphi_k}{2} \cdot \sum\nolimits_{M-(p-1)} \sin\left\{\frac{1}{2}(\varphi_2 + ... + \varphi_{2M+1}) - (\varphi_2 + ... + \varphi_{2M+1})_{M-(p-1)}\right\} \quad (59)$$

It is follows that:

$$E_{2M+1}^{(M+1)} = (-1)^{\frac{M(M-1)}{2}} 2^{2M(M-1)+1} \left(\prod_{j=k+1}^{2M+1}\prod_{k=2}^{2M} \sin\frac{\varphi_j - \varphi_k}{2}\right) \sin\frac{\varphi_2 + \varphi_3 + ... + \varphi_{2M+1}}{2} \quad (60)$$

The following co-determinant is:

$$E_{2M+1}^{(M+2)} = (-1)^{\frac{M(M-1)}{2}} 2^{2M(M-1)+1} \prod_{j=k+1}^{2M+1}\prod_{k=2}^{2M} \sin\frac{\varphi_j - \varphi_k}{2} \cdot \sum\nolimits_{M-1} \cos\left\{\frac{1}{2}(\varphi_2 + ... + \varphi_{2M+1}) - (\varphi_2 + ... + \varphi_{2M+1})_{M-1}\right\} \quad (61)$$

For every $3 \le p \le M$ we can write that:

$$E_{2M+1}^{(M+p)} = (-1)^{\frac{M(M-1)}{2}} 2^{2M(M-1)+1} \prod_{j=k+1}^{2M+1}\prod_{k=2}^{2M} \sin\frac{\varphi_j - \varphi_k}{2} \cdot \sum\nolimits_{M-(p-1)} \cos\left\{\frac{1}{2}(\varphi_2 + ... + \varphi_{2M+1}) - (\varphi_2 + ... + \varphi_{2M+1})_{M-(p-1)}\right\} \quad (62)$$

For every $2 \le p \le M-1$ it follows that:

$$E_{2M+1}^{(2M)} = (-1)^{\frac{M(M-1)}{2}} 2^{2M(M-1)+1} \left(\prod_{j=k+1}^{2M+1}\prod_{k=2}^{2M} \sin\frac{\varphi_j - \varphi_k}{2}\right) \cos\frac{\varphi_2 + ... + \varphi_{2M+1}}{2} \quad (63)$$

The check of the general formula was done using the Matlab program package, which will be the topic of the following chapter of this paper. The solutions have been checked up to the *M*=25 order, since this is the most complex case that can be expected to occur in reality. All of this gives reasons to authors to accept that the offered solution is really of a general character. In Appendix A we give the form of solutions of obtained determinants of fifth order (*M*=2), to help to understand the record of derived results.

From the obtained result, it is clear that it is not necessary to use the standard procedure for solving the system of equations, as suggested in [4]. This procedure in the case of an extremely complex spectral content of a signal would require a powerful processor and enough time for processing. Through the given expression, which is practically an on-line one for the known frequency spectra of input signals, we determine all of the unknown parameters (amplitudes and phases)

By reducing the resolution, the process of reconstruction is speeded up, while the concept of the double integration gets closer to the concept of the operation of the sigma-delta AD converter. We can conclude that proposed concept of processing without any change and limitation can apply in operation with sigma-delta ADC, enabling with this high resolution and speed in processing of AC signals. From the obtained result, it is clear that it is not necessary to use the standard procedure for solving the system of equations, as suggested in [4]. This procedure in the case of an extremely complex spectral content of a signal would require a powerful processor and enough time for processing. Through the given expression, which is practically an on-line one for the known (expected) spectral content, we determine all of the parameters.

Based on the suggested concept of processing, the complete realization of the necessary hardware can be achieved through the resources of the microprocessor itself, as well as through its counting resources. All these elements will lead to reduced costs of the suggested solution. It is assumed that, during these observations, the system remains stationary (electric utilities), and that the harmonics content is preserved, within the time interval necessary to conduct the measuring according to the suggested method [2].

3. SIMULATION RESULTS

Additional testing of the realized calculations was carried out by simulation in the program package Matlab (version 7.0). The following Table 1 shows the results obtained through the application of the suggested procedure in program package Matlab, for a different harmonic content of the input voltage signals. The results obtained by applying the relations (53) and (56)-(63) were compared with the results, which, as a solution to the system, i.e. the determinant, is offered in the Matlab program package itself.

The values in columns separated with commas correspond to harmonics and determinants with different orders. This means that for example in the first column for A_k values 0.0056; 0.00258; -3.32e-4; -0.00151; correspond to A_1=0.0056; A_2=0.00258, A_3=-3.32e-4, A_4=-0.00151, in the first column for $E_{2M+1}^{(l)}$ values 0.00695; 0.01096; 0.00433; 0.00123; 0.00043; 0.00399; 0.00516; 0.00212; 0.00007; correspond to $E_{2M+1}^{(1)}$=0.00695; $E_{2M+1}^{(2)}$=0.01096; $E_{2M+1}^{(3)}$=0.00433; $E_{2M+1}^{(4)}$=0.00123; $E_{2M+1}^{(5)}$=0.00043; $E_{2M+1}^{(6)}$= 0.00399; $E_{2M+1}^{(7)}$=0.00516, $E_{2M+1}^{(8)}$=0.00212; $E_{2M+1}^{(9)}$=0.00007 and so on. In Table 1, notation is completely in agreement with equations (2) and (6). The simulation results indicate that the performance improvement of signal reconstruction is remarkable.

This procedure can be used for the spectral analysis as well, where it is possible to find out the amplitude and the phase values of the signal harmonic, based on the set (predicted) system of equations. In case that there are non-harmonic components to the signal, it is necessary to identify their frequencies, and then apply the proposed procedure.

Although very widely used, the DFT algorithm has certain limitations in its application to harmonic analysis in power systems. There are circumstances in which it is not easy to distinguish between valid and spurious components. Magnitude errors arise. Phase errors can be substantial. By comparison, the method developed in this paper overcomes fully these limitations. It is free from spectral leakage errors. While the computational load of the iterative step involving FFTs does not change with the number of sampling values for some of the non-matrix implementation, the speed of convergence is improved if more points are available. This also corresponds to the natural intuition that the knowledge of more sampling values should lead to better convergence. Quite the contrary happens for some of matrix methods, as we proposed here. A larger number of sampling points or a large spectrum results in large matrices. Therefore the computational load for matrix methods (either iterative or those using pseudo-inverse matrices) increases quickly. Thus they may be extremely efficient for the situation with few sampling points, but fairly slow if there are many sampling points. This is the main reason why we think that our derived analytical solution is more computationally attractive for moderately sized problems and even makes it feasible for large reconstruction problems.

Table 1. *Results of a simulation of the derived expression for solving a system of equations with which the reconstruction of the observed signal is done*

Simulation number	1	2
A_0	0.007	0.007
A_k	0.0056;0.00258; -3.32e-4; -0.00151	0.0056;0.00258;-3.32e-4; -0.00151; -9.0032e-4; 3.279e-4
φ_k	π/9; 2π/9; 3π/9; 4π/9; 5π/9; 6π/9; 7π/9; 8π/9; π	π/13; 2π/13; 3π/13; 4π/13; 5π/13; 6π/13; 7π/13; 8π/13; 9π/13; 10π/13; 11π/13; 12π/13; π
E_{2M+1} determined by standard procedure	1.2351e-6	-3.974e-9
E_{2M+1} determined based on suggested procedure	1.2351e-6	-3.974e-9
$E_{2M+1}^{(l)}$ determined by standard procedure	0.00695; 0.01096; 0.00433; 0.00123; 0.00043; 0.00399; 0.00516; 0.00212; 0.00007	-3.03e-7; -5.21e-7; -1.72e-7; -1.45e-7; -6.06e-8; -6.44e-9; -2.19e-9; -1.28e-7; -3.28e-7; -1.29e-7; -4.18e-8; -1.697e-8; -2.66e-10
$E_{2M+1}^{(l)}$ determined based on suggested procedure	0.00695; 0.01096; 0.00433; 0.00123; 0.00043; 0.00399; 0.00516; 0.00212; 0.00007	-3.03e-7; -5.21e-7; -1.72e-7; -1.45e-7; -6.06e-8; -6.44e-9; -2.19e-9; -1.28e-7; -3.28e-7; -1.29e-7; -4.18e-8; -1.697e-8; -2.66e-10

For the proposed procedure, the problem of synchronization is expressed a much less than with the standard techniques of synchronized sampling, since this is a case of a real reconstruction of a signal, i.e. determining its basic parameters (phase and amplitude). The operation of integration must be done as many times as there are harmonic components in the processed signal, i.e. if the number of the harmonic components is M, it is necessary to perform $2M+1$ integer operations, in order to be able to start the reconstruction of the signal following the proposed method. It is necessary to have the information regarding the frequency of the fundamental harmonic in order to be able to determine the parameters of the system determinants.

4. CONCLUSION

The realized algorithm is a promising approach for determining the AC signal reconstruction. Our proposed implementations make this algorithm significantly more computationally attractive for moderately sized problems and even make it feasible for large reconstruction problems. The derived analytical expression opens a possibility to perform on-line calculations of the basic parameters of AC signals (the phase and the amplitude), while all the necessary hardware resources can be satisfied by a simple microprocessor or a signal processor of standard features. What has been avoided here is the use of a separate sample-and-hold circuit which, as such, can be source of a system error. The suggested concept can be used as separate algorithm as well, for the spectral analysis of the processed signals. Based on the identified parameters of the voltage and current signals, we can establish all the relevant values in the electric utilities (energy, power, RMS values).

5. REFERENCES

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APPENDIX A

For the fifth order (M=2) derived relations for determinats of system (5) have the form:

$$E_5 = -2^8 \sin\frac{\varphi_2-\varphi_3}{2}\sin\frac{\varphi_2-\varphi_4}{2}\sin\frac{\varphi_2-\varphi_5}{2}\sin\frac{\varphi_3-\varphi_4}{2}\sin\frac{\varphi_3-\varphi_5}{2}\sin\frac{\varphi_4-\varphi_5}{2} \quad (64)$$

$$E_5^{(1)} = -32 \sin\frac{\varphi_2-\varphi_3}{2}\sin\frac{\varphi_2-\varphi_4}{2}\sin\frac{\varphi_2-\varphi_5}{2}\sin\frac{\varphi_3-\varphi_4}{2}\sin\frac{\varphi_3-\varphi_5}{2}\sin\frac{\varphi_4-\varphi_5}{2} \left(\cos\frac{\varphi_2+\varphi_3-\varphi_4-\varphi_5}{2}+\cos\frac{\varphi_2+\varphi_4-\varphi_3-\varphi_5}{2}+\cos\frac{\varphi_2+\varphi_5-\varphi_3-\varphi_4}{2}\right) \quad (65)$$

$$E_5^{(2)} = -32 \sin\frac{\varphi_2-\varphi_3}{2}\sin\frac{\varphi_2-\varphi_4}{2}\sin\frac{\varphi_2-\varphi_5}{2}\sin\frac{\varphi_3-\varphi_4}{2}\sin\frac{\varphi_3-\varphi_5}{2}\sin\frac{\varphi_4-\varphi_5}{2} \cdot \left(\sin\frac{\varphi_4+\varphi_5+\varphi_3-\varphi_2}{2}+\sin\frac{\varphi_4+\varphi_5+\varphi_2-\varphi_3}{2}+\sin\frac{\varphi_2+\varphi_3+\varphi_4-\varphi_5}{2}+\sin\frac{\varphi_2+\varphi_3+\varphi_5-\varphi_4}{2}\right) \quad (66)$$

$$E_5^{(3)} = -32 \sin\frac{\varphi_2-\varphi_3}{2}\sin\frac{\varphi_2-\varphi_4}{2}\sin\frac{\varphi_2-\varphi_5}{2}\sin\frac{\varphi_3-\varphi_4}{2}\sin\frac{\varphi_3-\varphi_5}{2}\sin\frac{\varphi_4-\varphi_5}{2}\sin\frac{\varphi_2+\varphi_3+\varphi_4+\varphi_5}{2} \quad (67)$$

$$E_5^{(4)} = -32 \sin\frac{\varphi_2-\varphi_3}{2}\sin\frac{\varphi_2-\varphi_4}{2}\sin\frac{\varphi_2-\varphi_5}{2}\sin\frac{\varphi_3-\varphi_4}{2}\sin\frac{\varphi_3-\varphi_5}{2}\sin\frac{\varphi_4-\varphi_5}{2} \cdot \left(\cos\frac{-\varphi_2+\varphi_3+\varphi_4+\varphi_5}{2}+\cos\frac{\varphi_2-\varphi_3+\varphi_4+\varphi_5}{2}+\cos\frac{\varphi_2+\varphi_3-\varphi_4+\varphi_5}{2}+\cos\frac{\varphi_2+\varphi_3+\varphi_4-\varphi_5}{2}\right) \quad (68)$$

$$E_5^{(5)} = -32 \sin\frac{\varphi_2-\varphi_3}{2}\sin\frac{\varphi_2-\varphi_4}{2}\sin\frac{\varphi_2-\varphi_5}{2}\sin\frac{\varphi_3-\varphi_4}{2}\sin\frac{\varphi_3-\varphi_5}{2}\sin\frac{\varphi_4-\varphi_5}{2}\cos\frac{\varphi_2+\varphi_3+\varphi_4+\varphi_5}{2} \quad (69)$$