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REDUCING THE NUMBER OF FINITE ELEMENTS WHEN VERY THIN SUBDOMAINS ARE INVOLVED

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Abstract: *The paper proposes a method for reducing the number of nodes and triangular finite elements when solving an electromagnetic plan parallel field problem using 2D FEM (two dimension finite element method) and when very thin subdomains are involved. Vector Fields OPERA 2D program was used, but the proposed method can be applied to all types of 2D FEM computing programs.*

1. INTRODUCTION

The problem of finding the magnetic field distribution solution in iron plates appears when lifting magnets are used in order to move those packages of iron plates. The calculation of the force exerted by such a lifting magnet was the starting point of the present paper. Because between the iron plates appear very small air gaps, neglecting them in the calculation of the magnetic field distribution leads to huge errors in the results. The sketch (principle) of the electromagnet acting on several iron plates is presented in figure 1. In order to put in evidence the air layers their thickness is exaggerated in this drawing.

If we are taking into account this thin layers of air, as we should, because of their very small transversal dimensions, when FEM is used we need to work with a very large number of finite elements. Not only the thin air gaps must be covered with small triangles (finite elements), but also the adjacent surface of the iron plates must be discretized with similar small FE. This situation is not convenient because of the large amount of memory that is necessary on the computer and also because of the important increase of the computing time for solving the field distribution problem.

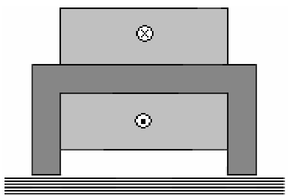

Figure 1. *The elevator electromagnet with the iron plates*

In this paper is proposed a method of taking into account the air gaps, but in an equivalent manner, so that the number of finite elements in the space of the thin air layers should be not so big.

The calculations will be made in a modified, but equivalent geometry. The modification of the geometry is made in order to obtain comparable dimensions (thickness) for the air gap as well as for the iron plates and, as a consequence, FE with comparable size can be used for the discretization of both regions. In the same time the equivalence must assure the same value for the computed lifting force of the electromagnet.

2. THE PRINCIPLE OF THE METHOD

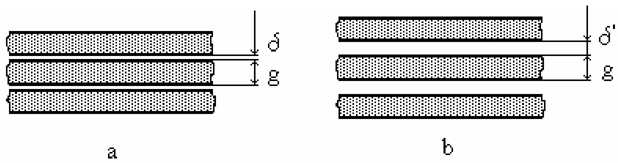

Figure 2. *The real geometry (a) and the equivalent one (b)*

In figure 2a the iron plates are represented with their real thickness “g” and the real air gap of “δ” dimension (very small; $\delta << g$). In order to solve the magnetic field distribution it is proposed the equivalent situation shown in figure 2b. The equivalent “air gap” thickness is assumed to be δ', greater than δ, and big enough to permit a convenient discretization in finite elements.

Because the field in the iron plates must remain the same and the energy stored in the field also, in the equivalent situation the spaces between the plates are no longer air gaps (μ_0), but it is assumed that those spaces are filled with a material that leads to the same magnetic reluctance as the real air gaps. The equivalence for the magnetic reluctance was considered for the vertical field (R_{mv}) as well as for the horizontal field (R_{mh}) as shown in the figures 3 and 4.

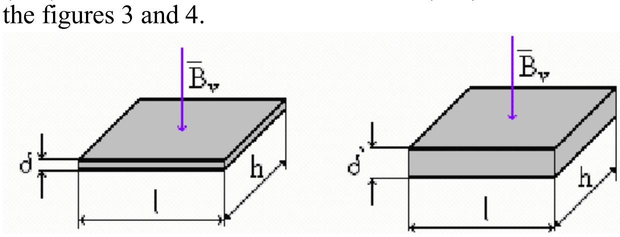

Figure 3. *The equivalence for vertical magnetic reluctance*

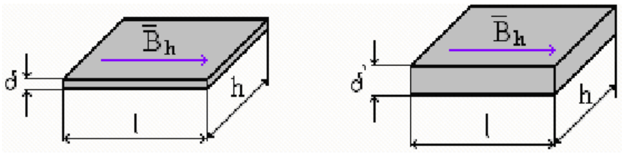

Figure 4. *The equivalence for horizontal magnetic reluctance*

In the real situation and in the equivalent one all the dimensions are the same, except the thickness δ which becomes δ'> δ. By this equivalence the values of the vertical magnetic permeability (μ_v) and of the horizontal one (μ_h) are obtained as in (1) and (2).

$$R_{mv} = \frac{\delta}{\mu_0 lh} = \frac{\delta'}{\mu_v lh} \quad ; \quad R_{mh} = \frac{l}{\mu_0 \delta h} = \frac{l}{\mu_h \delta' h} \qquad (1)$$

$$\mu_v = \mu_0 \frac{\delta'}{\delta}; \quad \mu_h = \mu_0 \frac{\delta}{\delta'} \qquad (2)$$

Now the air gap is substituted with a magnetically equivalent domain, with magnetic properties given by (2).

The same principle can be used when 2D current flow problem is solved and the electric conduction takes place through very thin homogenous conductive layers of σ conductivity. In this case the electric resistance must be the same. With the same type of geometry, as that presented in fig.3 and fig.4, the equivalent conductivities are given by (4):

$$R_v = \frac{\delta}{\sigma lh} = \frac{\delta'}{\sigma_v lh} \quad ; \quad R_{mh} = \frac{l}{\sigma \delta h} = \frac{l}{\sigma_h \delta' h} \qquad (3)$$

$$\sigma_v = \sigma \frac{\delta'}{\delta}; \quad \sigma_h = \sigma \frac{\delta}{\delta'} \qquad (4)$$

Also if an electrostatic problem is solved by 2D FEM and very thin air layers are involved the equivalence is

$$\varepsilon_v = \varepsilon_0 \frac{\delta'}{\delta}; \quad \varepsilon_h = \varepsilon_0 \frac{\delta}{\delta'} \qquad (5)$$

In conclusion, if the very small dimension δ is made "n" times bigger, δ'= n· δ, the equivalent material property in the direction of δ (permeability, conductivity or permittivity) is "n" times bigger also. In the orthogonal direction the property shall be "n" times smaller as that one from the real (homogenous) situation.

3. APPLICATION

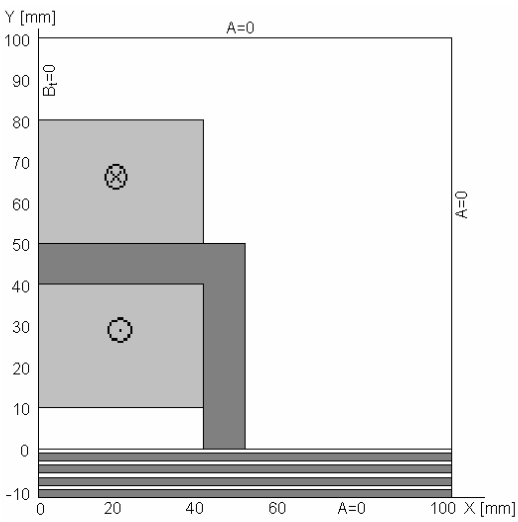

Fig. 5 *The domain for the field calculation, based on symmetry*

In order to verify the validity and the accuracy of the proposed method it is evaluated a very long lifting magnet, type U. The cross section is presented in Fig. 1 and the domain for the field calculation (reduced, due to the symmetry of the device) is presented in figure 5. Because of the very long length of the lifting magnet (dimension orthogonal on the plane of Fig. 1) the field can be assumed as being plan - parallel.

In this example the package of iron plates contains a number of 4 plates of g = 2 mm thickness each and air gaps of δ = 0.1 mm.

The iron U shape ferromagnetic material is nonlinear and the B-H curve is taken from the library of Vector Fields OPERA.

The material of the 4 iron plates is OL 37.

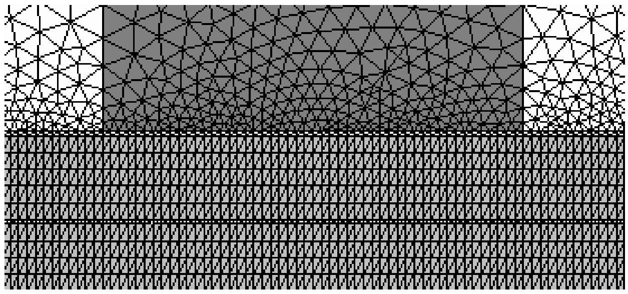

Figure 6 *The discretization of the real geometry in the pole zone*

Figure 6 shows the discretization of a part of the domain, containing the magnetic pole, two modified air-gaps and two iron plates.

The number of finite elements in the iron plates package is 24,000, in the case of modelization of the real situation with very thin air-gaps, even if the ratio of the cathetus of the FE triangles is 2.5 (not so convenient value). If the value of this ratio would be around a more suitable value of 1, the number of the triangles would be around 60,000.

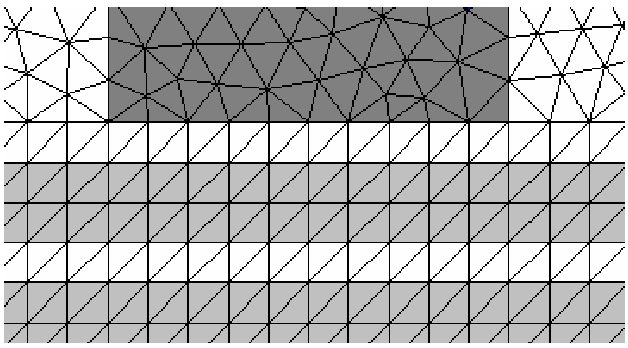

Figure 7 *The discretization of the equivalent geometry*

The equivalent geometry was made for δ'/δ=10, so that the absolute values of the permeability of the equivalent "air gaps" are:

$$\mu_v = \mu_y = 10\mu_0 \quad ; \qquad \mu_h = \mu_x = \frac{1}{10}\mu_0 \qquad (6)$$

For the equivalent geometry, with some details (zoom) shown in Fig.7, the number of finite elements in the iron plate package is only 2,400, even if this time we imposed a more convenient shape to the triangles.

For the steady-state situation we accepted an equivalent current density , uniformly distributed on the coil section, of J_{ech} = 1.5 A/mm^2 and the MMF, N·I is 1800 Amperes, N being the number of turns and I the current passing through the section of the coil's conductor.

Assuming a reasonable space factor of k=0.75 and with N=300 turns, the cross section of the cylindrical conductor of the coil results s_c = 3 mm^2.

For wires made on aluminum the resistivity of the material is about $\rho_{Al} = 3 \cdot 10^{-8}\ \Omega \cdot m$.

Assuming that the length of the coil (dimension orthogonal on the section shown in Fig.5) is 1m, for the electric resistance of the coil we obtain a value of R = 6 Ω.

The value of the current density trough the section of the wires is $J = J_{ech}/k$ = 2 A/mm^2.

The current is $I = J \cdot s_c$ = 6 A and the voltage U=R·I=36 V.

These values were used when solving the steady state situation.

It was solved also the transient regime problem. Applying the 36 V d.c. source to a R-L circuit we have obtained also the time variation for the current in this parametric circuit, with time variation of the inductance.

With these data was solved the problem of the magnetic field distribution in the two situations. The results for the real modelized situation and the equivalent modelized situation were compared and the results are presented in the followings.

4. RESULTS

A. The Computing Time

The transient regime produced by applying a step voltage of 36V was solved for a time extension of 1.5s (practically until the steady-state situation is completely established).

The results corresponding to the field distribution, as well as the current's values were stored for the following time moments: 0.01; 0.02; 0.05; 0.1; 0.15; 0.2; 0.3; 0.4; 0.5; 0.7; 1; 1.5s.

The two equivalent modelized situations will be named:

-air gap, real situation, called AG

-equivalent, "nonisotropic layer", called EQ

The computing time in the two modelized situations were very different. For the transient regime analyze this difference is more important: t_{AG} = 4 hours = 240 minutes and t_{EQ} = 11 minutes.

By the equivalent modelization, reducing the number of the finite elements, the time reduction has a spectacular reduction of about 22 times ($t_{AG}/t_{EQ} \approx 22$).

B. The Magnetic Flux Density Values Comparison

TABLE 1.VALUES OF MAGNETIC FIELD DENSITY B[T]

		Air Gap	Equivalent
	x=10mm;y=45mm	1.7996	1.7995
	x=45mm;y=20mm	1.6643	1.6640
x = 10mm at a distance 0.5mm inside the four plates	P_1	2.0345	3.0344
	P_2	2.0344	2.0343
	P_3	2.0343	2.0342
	P_4	2.0343	2.0342
x = 45mm at a distance 0.5mm inside the iron plates	P_1	1.3152	1.3246
	P_2	1.2039	1.2015
	P_3	1.3856	1.3874
	P_4	1.4539	1.4553

If the computing time is largely reduced maybe we should be concerned about the accuracy and concordance of the two sets of results.

It can be observed from Table 1 that the values are, by technical point of view, identical.

C. The Time Variationof the Current, i(t)

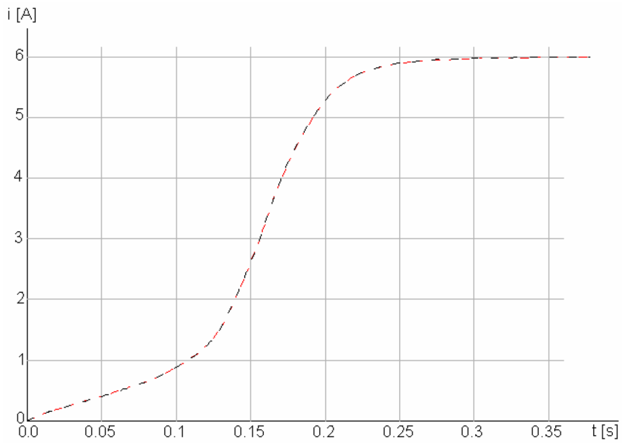

Figure 8.*The time variation of the current*

In Fig.8 are represented both currents: the one obtained for the air gap modelized situation and the one obtained for the equivalent thin layer modelization. They are practically identical.

D. The Picture of the Magnetic Field

In Fig.9 and Fig.10 are represented the pictures of the magnetic field for the air gap and for the equivalent nonisotropic layer, respectively. It can be seen that the magnetic field in the magnetic circuit of the lifting electromagnet (U shape), as well as in the iron plates are identical.

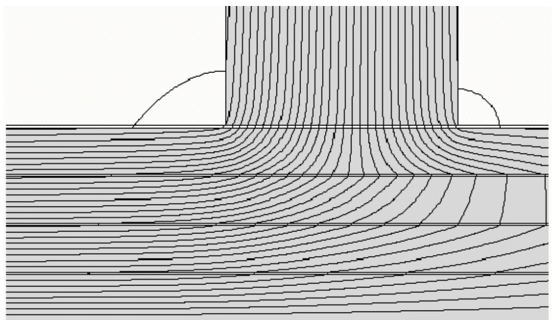

Figure 9 *The real air gap modelized situation*

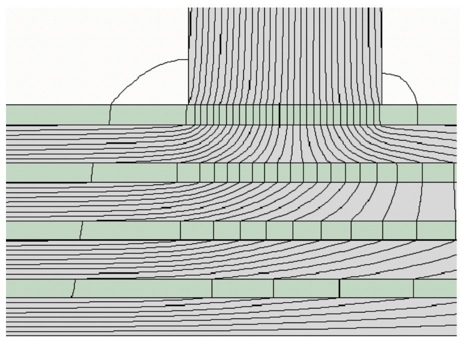

Figure 10 *The equivalent layer modelized situation*

The local values are different in the real air gaps and in the equivalent layers, but the integral values, for some important quantities such as the magnetic flux trough identical surface or the magneto motive force (MMF) on similar loops are identical and shall be presented further.

E. The Magnetic Flux at the Upper Surface of the Iron Plates

The magnetic flux on the unit length is:

$$\Phi^* = \frac{\Phi_s}{l}; \quad \Phi_s = \int_s \overline{B} \cdot \overline{dS} \tag{7}$$

The integration surface S represents the upper surface of the iron plates package. The length l is the dimension orthogonal on the section used in solving in 2D. The results are presented in Table 2.

TABLE 2. VALUES OF MAGNETIC FLUX Φ^*[mWb/m]

	P_1	P_2	P_3	P_4
Air Gap	16.29792	12.22195	8.14665	4.07146
Equivalent	16.27334	12.22108	8.14612	4.07120

F. The Magnetomotive Force calculated for two similar closed loops in the two modelized situations

In order to verify the solution we have calculated the MMF along a closed loop, a-b-c-d-a, aplying the Ampère's Circuital Law .

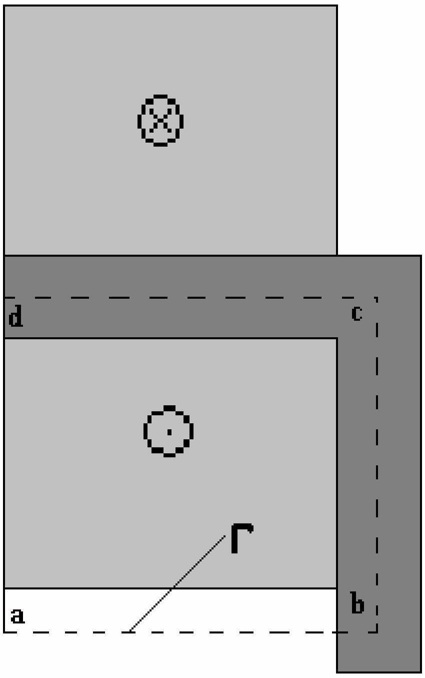

Figure 11. *The loop Γ for the verification of the Ampère's Law*

For the closed loop Γ shown in Fig.10, the coordinates of the points (see Fig.4) are the followings: a(0,5);b(45,5);c(45,45);d(0,45).

$$\oint_\Gamma \overline{H} \cdot \overline{dl} = \int_{S_\Gamma} \overline{J} \cdot \overline{dS}; = N \cdot I = 1800 \text{ A} \tag{8}$$

Using the components of the magnetic field, calculated with Vector Field OPERA 12.0, the integral (8) was calculated as shown in (9).

$$\oint_\Gamma \overline{H} \cdot \overline{dl} = \int_a^b H_x \cdot dx + \int_b^c H_y \cdot dy - \int_c^d H_x \cdot dx \tag{9}$$

In (9) was taken into account that: $\int_d^a H_y \cdot dy = 0$

TABLE 3
VERIFICATION OF THE AMPÈRE'S CIRCUITAL LAW

	$\int_a^b H_x \cdot dx$	$\int_b^c H_y \cdot dy$	$-\int_c^d H_x \cdot dx$	$\oint_\Gamma \overline{H} \cdot \overline{dl}$
Air Gap	1313.348 A	138.261 A	347.181 A	1798.8 A
Equivalent	1313.99 A	137.872 A	346.964 A	1798.8 A

From the Table 3 we can see not only that the EMF along Γ curve is practically identical for the two modelized situation, but also that it is practicaly equal to the theorethic value of 1800 A.

5. CONCLUSIONS

The method proposed for improving the discretization of the 2D domains for FEM can be applied for different types of electromagnetic field distribution. In this paper only the plan paralel field was analysed, but this method can be applied also for plan meridian field, with different equivalation relations.

The accuracy of the solution is very good, in both situations, but the geometry with the equivalent layer has a much smaller number of finite elements. This smaller number of nodes and finite element implies an important reduction in the computing time for solving the field distribution.

6. REFERENCES

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[4] *Vector Fields OPERA 2D User Guide*, Version 12, May 2008