15th INTERNATIONAL SYMPOSIUM on POWER ELECTRONICS - Ee 2009
XV Međunarodni simpozijum Energetska elektronika – Ee 2009
NOVI SAD, REPUBLIC OF SERBIA, October 28th - 30th, 2009

CONTROLLER DESIGN FOR A FREE GYROSCOPE SEEKER

R.Zaeim, M.A.Nekoui*
Electrical Eng. Dept Science and Research Branch, Azad University
* Electrical Eng. Dept K N T University of technology

Abstract: *In this paper we discuss and compare two methods to control the free gyroscope seeker scan loop system: centralized con-trol structure and decentralized control structure. First a centralized controller based on the linear model of the seeker is provided. We next proceed to design a decentralized controller using a non-linear model of the seeker. Input-Output Pairing is a vital stage in decentralized control and since the seeker model that is used in this structure is nonlinear, nonlin-ear RGA is used for input-output pairing analysis. SISO controllers are designed and proposed to track the specific scan patterns like conical and rosette.*

Key Words: ***centralized control, decentralized control, Free Gyroscope Seeker.***

1. INTRODUCTION

The seeker is mounted in a missile head and is used to search for a target by following a given scan pattern. One of the most popular seekers is the Free Gyroscope Seeker (FGS), where it is modeled in two forms: linear and nonlinear multivariable plant [1,2].

In both of these models there exist undamped oscillations. In many applications, specifically in the seeker of homing missile, free gyroscope will be useful only when its un-damped oscillations become attenuated[3,4].

The goal of this paper is to specify the manner of controlling the FGS such that the seeker scan loop can search and track a scan pattern, conical or rosette, with minimum error. The torques which are applied to the rotor are introduced as control inputs and its angular rate of turn about the rotor axis will be the control output.

There are various methods to control the linear multivariable plants. In this paper we design centralized controllers using the High Gain PI controller [5] for the linear model of FGS. But, the main model of free gyroscope is nonlinear and in the next step we propose the decentralized control structure to control it. Because, in this structure we use SISO controllers, like PID to control the plant and this category of controllers can solve the control problem and deal with nonlinearity and uncertainty in the model. But, in control structure design for decentralized control, input-output pairing is a vital stage and we cannot use popular methods like Relative Gain Array or other input-output pairing methods, which are introduced for linear multivariable plants. Hence, we will use the recent nonlinear-RGA [6] and analyze the result.

In section 2 a short survey on linear and non-linear models for free gyroscope seeker (FGS) is given. Section 3 provides centralized control for FGS. In section 4, we propose the decentralized control and the input-output pairing analysis is developed. In section 5, a real FGS model is considered and using the results of decentralized control structure, SISO controllers are designed to show the effectiveness of the method.

2. DYNAMICAL MODELING OF FREE GYROSCOPE SEEKER

Dynamical modeling of FGS is mentioned in different dynamics references. Dynamic equations of FGS are derived in [1,2] and are represented as follows:

$$\begin{aligned} T_x &= -I_R \mathrm{Cos}\theta_D \dot{\psi}_D \dot{\theta}_D \\ &\quad - I_R \mathrm{Sin}\theta_D \ddot{\psi}_D + I_R \dot{\omega}_s \\ T_y &= I_r \ddot{\theta}_D \ + I_R \omega_S \mathrm{Cos}\theta_D \dot{\psi}_D \\ &\quad -\left(I_R - I_r\right)\mathrm{Sin}\theta_D \mathrm{Cos}\theta_D \left(\dot{\psi}_D\right)^2 \\ T_z &= I_r \mathrm{Cos}\theta_D \ddot{\psi}_D \ - I_R \omega_S \dot{\theta}_D \\ &\quad + \left(I_R - 2I_r\right)\mathrm{Sin}\theta_D \dot{\psi}_D \dot{\theta}_D \end{aligned} \tag{1}$$

Where ψ_D and θ_D refer to the Euler rotating angles about reference z_D axis and y_D axis in Direct coordinate frame D respectively. I_R and I_r are moment of inertia. T_y and T_z are torques that act on rotors along the reference y axis and z axis and ω_s is the angular velocity about reference x axis. Using the state variables $x_1 = \psi_D, x_2 = \dot{\psi}_D, x_3 = \theta_D, x_4 = \dot{q}_D, x_5 = w_s$, manipulated variables $u = [T_y \ \ T_z]^T$ and the measured variables $y = [x_3 \ \ x_1]^T$, the nonlinear state space model of free gyroscope is obtained as in (2). Bear in mind some assumptions like the angular velocity is rotated at a constant rate ω_s , also angular velocity ω_s is much larger than $\ddot{\psi}_D, \dot{\psi}_D, \dot{\theta}_D, \ddot{\theta}_D$ and the torque T_x is zero. Now a simple model of the free gyroscope can be obtained using these assumptions as in (3), where $H = I_R \omega_s$ is the angular momentum. Also linear equations of FGS can be presented as (4),[2].

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \left[(T_z + I_R x_4 x_5)/I_r \cos x_3\right] \\
&\quad -\left[((I_R - 2I_r)x_2 x_4 \sin x_3)/I_r \cos x_3\right] \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= \left[(T_y - I_R x_2 x_5 \cos x_3)/I_r\right] \\
&\quad +\left[((I_R - I_r)x_2^2 \sin x_3)/I_r\right] \\
\dot{x}_5 &= \left[(x_2 x_4 \cos x_3)\right] \\
&\quad +\sin x_3 \left[(T_z + I_R x_4 x_5)/I_r \cos x_3\right] \\
&\quad -\left[(I_R - 2I_r)x_2 x_4 \sin^2 x_3 / I_r \cos x_3\right]
\end{aligned}
\tag{2}
$$

$$
\begin{aligned}
T_x &= 0 \\
T_y &= H \cos\theta_D \dot{\psi}_D \\
T_z &= -H\dot{\theta}_D
\end{aligned}
\tag{3}
$$

$$
\begin{aligned}
T_y &= I_r \dot{\omega}_y + I_R \omega_s \omega_z \\
T_z &= I_r \dot{\omega}_z - I_R \omega_s \omega_y
\end{aligned}
\tag{4}
$$

Where ω_y and ω_z are angular velocities of Direct coordinate frame D about reference y_I axis and z_I axis of reference coordinate frame *I*.

the transfer function model of free gyroscope is

$$
\begin{bmatrix} \omega_y \\ \omega_z \end{bmatrix} = \frac{1}{s^2 + \omega_N^2} \begin{bmatrix} s/I_r & \dfrac{-I_R\omega_s}{I_r^2} \\ I_R\omega_s/I_r^2 & s/I_r \end{bmatrix} \begin{bmatrix} T_y \\ T_z \end{bmatrix}
\tag{5}
$$

Where ω_z ω_y are the angular velocities about reference z axis and y axis respectively and $\omega_N = (I_R / I_r)\omega_s$.

3. CENTRALIZED CONTROLLER

There are diverse methods to control the linear multivariable plants, which can stabilize the linear plant. In this section we attempt to achieve our goals mentioned in the introduction by designing a "High Gain PI Controller" for linear model. we achieve that in centralized form.

Consider the MIMO plant described by the following state space equations:

$$
\begin{aligned}
\dot{x} &= Ax + Bu \\
y &= Cx
\end{aligned}
\tag{6}
$$

Assume the rank of *B* to be *m*. Equation (6) can be rewritten as (7)

$$
\begin{aligned}
\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} &= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 \\ B_2 \end{bmatrix} u \\
y &= \begin{bmatrix} C_1 & C_2 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}
\end{aligned}
\tag{7}
$$

Where $X_2(t)$ is an m-dimensional matrix.

The MIMO plant, given by equation (7) will be regular if the plant is minimum-phase and matrix $C_2 B_2$ be full rank. In this case a stable plant without interaction is achievable by using multivariable "High Gain PI Controller".

If matrix $C_2 B_2$ loses rank, then the plant is called irregular [5], like the linear model of gyroscope. For controlling such irregular plants, we use an internal loop feedback as shown in Figure 1.

In figure(1) it is assumed that state space equations are like (7) and a new output is define as $w(t) = y(t) + M\dot{x}_1(t)$.

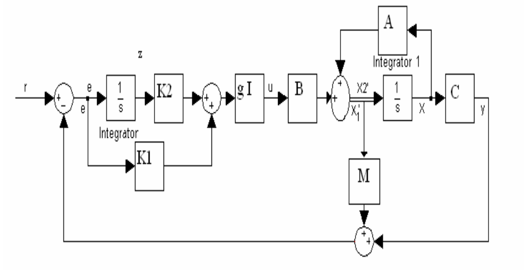

Fig. 1: *closed loop system with high gain PI controller for irregular plant*

The applied control law is

$$
u(t) = g\{k_1 e(t) + k_2 z(t)\}
\tag{8}
$$

Where *g* is the scalar gain, *e(t)=r(t)-y(t)* is the error vector, signal w is $W(t) = y(t) + M\dot{X}_1(t)$ and z is the integral of the error vector. With appropriate choice of K_1 and K_2 we can put the eigenvalues of system in desired place. one choice is $K_2 = -\lambda_0 K_1$, $K_1 = (CB)^{-1}S$, and $S = diag\{s_1 \ \ldots \ s_m\}$, $s_i \in \Re^+$.

Now the eigenvalues of closed loop system are placed in $-g\sigma_i$. Appropriate choice of K_1, K_2, M, l_0 and g can stabilize the linear model.

Figure 4 shows the step responses of the linear FGS model(ψ_D and θ_D) using proposed centralized control when T_y=8Nm is applied at t=0s and T_z=12Nm at t=1.5s. As seen in Figure 4, there exists some low interaction.

4. DECENTRALIZED CONTROLLER

One of the most practical methods to control the multivariable plants is decentralized control. While centralized multivariable controllers are complex and lack integrity, the decentralized control system enjoys certain advantages: 1- it requires fewer parameters to tune which are easier to be understood and implemented; and 2- loop failure tolerance of the resulting control system can be assumed during the design phase [7]. As shown in (1), (2) the model introduced for FGS is nonlinear and there exists high coupling between its equations.

In this section we propose a decentralized control structure to control it.

4.1. Input-output pairing analysis

In decentralized control design, the primary task is loop configuration or input-output pairing to achieve the minimum interaction among loops. Since the seminal work by Bristol (1966) and presentation of the RGA concept for input-output pairing, there have been many extensions and modifications to the method (Khaki-Sedigh, 1996; Conley and Salgado, 2000; Wittemark and Salgado, 2002). In spite of the extensive research of the previous decades, the input-output pairing of nonlinear multivariable systems is an open problem and only it has been addressed[8],[9]. Moaveni and Khaki-Sedigh (2007) present a nonlinear RGA for affine nonlinear multivariable systems.

Moaveni and Khaki-Sedigh in [6] introduced the nonlinear RGA for input-output pairing of affine nonlinear multivariable systems as:

$$\begin{aligned} \dot{x} &= f(x) + \sum_{j=1}^{m} g_j(x)u_j \\ y_1 &= h_1(x) \\ &\vdots \\ y_m &= h_m(x) \end{aligned} \tag{9}$$

Where, nonlinear RGA is as follows

$\Gamma_{nl-RGA} \triangleq R \otimes R^{-T}$ And:

$$R = \begin{bmatrix} L_{g_1(x)}L_{f(x)}^{r-1}h_1(x) & \dots & L_{g_m(x)}L_{f(x)}^{r-1}h_1(x) \\ L_{g_1(x)}L_{f(x)}^{r-1}h_2(x) & \dots & L_{g_m(x)}L_{f(x)}^{r-1}h_2(x) \\ \vdots & & \vdots \\ L_{g_1(x)}L_{f(x)}^{r-1}h_m(x) & \dots & L_{g_m(x)}L_{f(x)}^{r-1}h_m(x) \end{bmatrix}$$

and, $r \leq n$ is the maximum value of vector relative degree from y_i to u_j, where, $i, j = 1,\dots,m$.

From equations (2) and (9) we know that FGS has an affine nonlinear multivariable model. So, we can use nonlinear RGA to choose the appropriate input-output pair for FGS. To compute the nonlinear RGA for FGS, $r = 2$ and matrix R is:

$$R = \begin{bmatrix} \frac{1}{I_r \cos x_3} & 0 \\ 0 & \frac{1}{I_r} \end{bmatrix} \tag{10}$$

So, nonlinear RGA is as (11) and it is important to note that $\cos x_3 \neq 0$, because always $-\frac{\pi}{2} < x_3 < \frac{\pi}{2}$ [1,2].

$$\Gamma_{nl-RGA} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{11}$$

According to nonlinear RGA, (11), $(u_1 - y_1, u_2 - y_2)$ is an appropriate pair on the whole of operational field of FGS and is independent of the operational points of the FGS.

4.2. Decentralized PD Controller

Controller structure design and the choice of an appropriate pair according to nonlinear RGA analysis, proposes a control loop configuration as in Figure 2.

After this stage we should design the SISO controllers to control the plant. To do this part of the design procedure, we first linearize the nonlinear model around the operational point, like (12), and then plot the bode diagram of the related subsystems to choose the appropriate controller, like Figure 3.

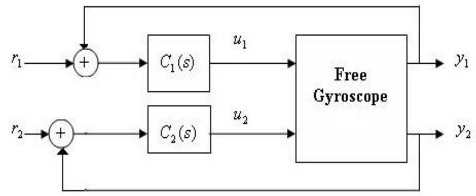

Fig.2 *Loop Configuration to Decentralized Control of Free Gyroscope Seeker*

$$G(s) = \begin{bmatrix} \frac{81633}{s^2 + 6316400} & -\frac{201 \times 10^6}{s(s^2 + 6316400)} \\ \frac{2.2796 \times 10^8}{s(s^2 + 6316400} & \frac{88890}{s^2 + 6316400} \end{bmatrix} \tag{12}$$

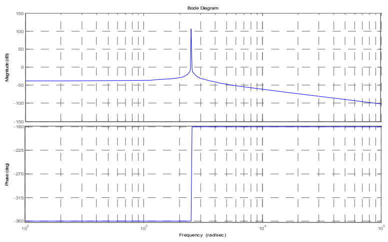

Fig.3 *Bode diagram of related subsystem*

According to the bode plot of the subsystem in Figure 3, we propose the PD controller as a SISO controller:

$C_j(s) = K_{pj} + K_{dj}s \quad j = 1,2$

Hence, the PD controller is as $(K_{pj}, K_{dj}) = (50,\ 0.0225)$ for $j = 1,2$. Figure 4 shows the step response of the nonlinear FGS model using the proposed decentralized control and introduced PD controller when T_y=8Nm is applied at t=0s and T_z=12Nm at t=1.5s.

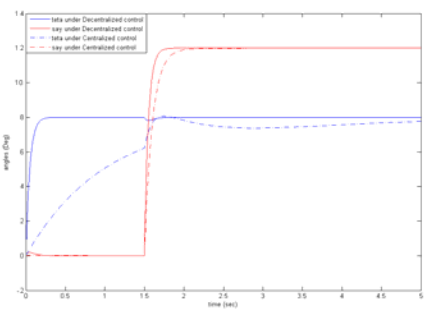

Fig.4. *Step Response of linear FGS under centralized Control and nonlinear FGS under decentralized control*

As shown in Figure 4, in decentralized control, interaction between inputs and outputs is completely attenuated.

5. SCANNING PERFORMANCES

There are two important scanning patterns as conical and rosette, which are designed to enlarge the field of view of the seeker. The reference inputs needed for generating these two patterns are obtained as follows:

- Conical scan pattern:

$$r_1 = a.\cos(q(t)) \qquad r_2 = \pm a.\sin(q(t)) \tag{12}$$

- Rosette scan pattern:

$$r_1 = 2a.\cos(3q(t)) - a.\cos(q(t)) \qquad r_2 = \pm[2a.\sin(3q(t)) + a\sin(q(t))] \tag{13}$$

Where a shows the instantaneous amplitude of the field of view and $\pm$ denotes the scanning direction.

The responses of purposed FGS to above scan patterns are shown in Figures 5 and 6. These responses and their performances are satisfactory, as the steady state error for conical pattern is less than 1% and for rosette pattern is less than 2%.

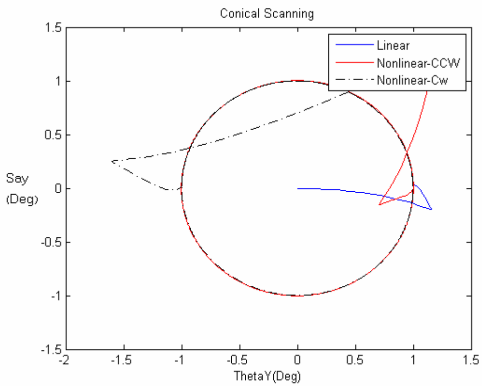

Fig.5 *Conical Scanning Performance for Decentralized PID controllers*

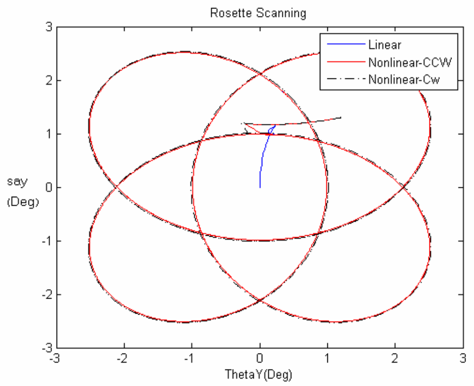

Fig.6 *Rosette Scanning Performance for Decentralized PID controllers*

6. CONCLUSION

In this paper we designed centralized and decentralized control structures for the Free Gyroscope Seeker. In centralized control, we design "High Gain PI Controller" using a linear model. Simulation results show that the interaction between input-output channels does not attenuate completely due to the linear model that is not a sufficiently detailed plant model and it is achieved by counteracting the interactions through the elaborate design of all the controller transfer function matrix elements

After centralized control, we proposed a decentralized control structure for FGS. In first step we chose the appropriate input-output pair using nonlinear RGA and it shows that $(u_1 - y_1, u_2 - y_2)$ is an appropriate pair and there is no chance to change the pair. Afterwards, single input-single output PID controllers are designed for the linearized model and those are employed to control the real FGS. Simulation results show that there is no more interaction between input-output channels and that each input acts just on one output. Finally, the effectiveness of the proposed methods is shown.

Note: in this paper, for different parameters of free gyroscope, we use real values related to a electro optical free gyroscope seeker

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