Paper No. T4-1.5, pp. 1-5

15th INTERNATIONAL SYMPOSIUM on
POWER ELECTRONICS - Ee 2009
XV Međunarodni simpozijum Energetska elektronika – Ee 2009
NOVI SAD, REPUBLIC OF SERBIA, October 28th - 30th, 2009

A CONTROL ENGINEER'S GUIDE TO HIGH PRECISION DIGITAL POSITION SYSTEMS SYNTHESIS

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Abstract: *This paper proposes a combined control system in synthesis of digitally controlled high-precision positional system. The designed system is based on both linear control algorithm and sliding-mode control (SMC) algorithm. A conventional PD controller is employed in the main control loop, whereas SMC system is implemented in the realization of so-called active disturbance estimator (ADE). The main controller provides positioning accuracy in nominal system. ADE compensates external and internal disturbances, forcing the real perturbed plant to behave as the nominal one. Signal differentiation and chattering are particular practical problems due to quantization noises that are introduced by digital realization and finite encoder resolution. Special attention is paid to minimization of the aforementioned unwanted effects. Theoretical analysis has been experimentally verified on a laboratory prototype of a positional system with DC motor.*

Keywords***: variable structure systems, quasi-sliding modes, digital sliding modes, integral control, PD position control.***

1. INTRODUCTION

Servo systems are widely used in many applications (machine tools, manipulators, robots, navigation systems etc.). They may be classified into two groups. The first group contains servo systems that are assigned to track motion trajectory of a moving object or to follow a command trajectory as accurate as possible. This class of the servo systems are named *tracking systems.* There are two different tasks: (i) to track an unknown in advance reference, and (ii) to track analytically known trajectory. The first task is more complicated, theoretically very difficult and represents a challenging problem for the practical realizations.

Servo systems that realize motions from point-to point without rigorous definition of moving trajectory belong into the second group. References in these systems are step type signals. The main task is to move as fast as possible from one position to the other, preferably without overshoot, regardless of the applied load and other disturbances. This class of servo systems is called *positional systems.*

Most of positional systems in mechatronics, robotics and various industrial applications are designed by using conventional PD controllers. As is well known, such systems can ideally track only constant signals if load is not applied. In the applications where accurate positioning is required these systems give unsatisfactory results. Some other control technique must be applied that provides great robustness.

One of solutions to the given control task may be implementation of sliding mode (SM) control systems (SMCS) [1], whose theoretical invariance to disturbances in ideal SM [2] is reduced to an excellent robustness in practical realizations. That is the reason why SMCS found their largest application exactly in this field. As a state space technique, SMCS need information of all state coordinates. This practically means the knowledge of the error signal and its successive derivatives.

In order to further improve system accuracy additional disturbance compensation is often carried out. Extraordinary improvements were achieved in [3], [4] in various servo systems by so-called active disturbance estimator (ADE), which contains a SM controlled active subsystem. Also, there is a possibility of introduction of supplemental integral action into SMCS that additionally increases system accuracy [5].

ADE is a tracking subsystem that controls the nominal plant model without disturbances. Therefore its all state coordinates are accessible. But, to obtain perfect tracking, it is necessary to have full information about its reference signal. Accordingly, this is possible only for the analytically known in advance references. This is the case only in positional systems not in the case of tracking systems. Tracking accuracy depends on a number of available derivatives of the tracking error signal [6]. Second order SM control is suggested in [7] for the servo system synthesis, where SM based differentiator is used for evaluation of the error signal derivative [8], [9]. Differentiators are practically useful only for the first and possibly for the second order derivatives of the signal, whereas high order derivatives are completely inapplicable due to severe noise contamination.

This paper proposes a way to upgrade conventional PD position systems into high-accuracy robust positioning systems by using ADE that compensates system disturbances and it is located in the local feedback loop. The ADE involves an active control substructure based on discrete-time SMC (DSMC). The proposed control extension is described in details, the DSM controllers design procedures are explained and the experimental tests on a DC motor are presented in the paper.

2. CONTROLLED PLANT MODEL

Until recently the high-precision positional systems were built up only by use of brush DC-motors (servomotors) or brushless DC-motors. Both type of motors are expensive, particularly brushless DC-motors. Novel constructions of high performance positional systems are based on three-phase squirrel cage induction motors (IMs) as cheap and reliable solution. These applications require vector control methods based on indirect rotor flux orientated control (IRFOC), [10], [11], [12]. Regardless of the type of applied motor, the controlled plant may be described by a unified mathematical model described by a second-order differential equation, if some inertial modes of the electrical subsystem are neglected being much faster then the mechanical one. The mathematical model of the plant in the state space is given as:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}u + \mathbf{d}f \qquad\qquad (1)$$
$$y = x_1$$
$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -0 & -a \end{bmatrix}; \mathbf{b} = \begin{bmatrix} 0 \\ b \end{bmatrix}; \mathbf{d} = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

where $\mathbf{x} = [x_1 \ \mathrm{x}_2]^{\mathrm{T}} = [\theta \ \omega]^{\mathrm{T}}$ - is vector of state, θ - is motor shaft position, ω - is velocity; u - is control, and f- is disturbance.

If DC motor is used, then the parameters in (1) are [5]:

$$a = 1/T_m; T_m = R_r J/c^2; b = K/cT_m;$$
$$f = \frac{M_o}{J} \qquad\qquad (2)$$

where: $T_m, c, R_r, J, K,$ and M_o, are respectively, mechanical time constant, universal motor constant, resistance of rotor loop, moment of inertia of motor with mechanical load, gain of motor with amplifier and load torque.

If an IM is used, the model (1) parameters are [4], [13]:

$$a = B/J, b = k_t/J\,, \qquad\qquad (3)$$

B- coefficient of viscous friction, k_t - is electromagnetic torque constant.

Remark 2.1 It is assumed that for positional system with IM the IFOC is applied with decoupling as well as compensation of rotor resistance variation.

Remark 2.2 The model (1) satisfies the matching condition [2].

3. CONVENTIONAL PD-TYPE POSITIONAL SYSTEM

Conventional PD-type positional system is usually based on cascade structure with inner velocity and current control loops [11]. The conventional approach for design of PD positional system is the method of magnitude optimum [11]. The essence of this method is to compensate mechanical time constant of the motor with differential time constant of the PD controller. The PD controller gain is chosen in such way that the system obtains desired coefficient of relative damping ζ. Since the mechanical time constant of the controlled motor may fluctuated in a wide range under load action, the desired ζ can be obtained only in nominal conditions.

For the PD controller design in discrete-time domain, there are two basic methods. If the sampling time is much smaller comparing to to the proces dynamics, design can be made in s-domain. After the obtained controller parameters in that domain, it is necessary to use one of digital redisign method (Tustin rule, for example). The second method is direct digital design by using pole placement method.

These methods are well known and will not be explained hire. To keep the controlled plant working in nominal conditions, an ADE should be used that will be explained in the folowing section.

4. ACTIVE DISTURBANCE ESTIMATOR

Information about the external disturbances is practically impossible to obtain by direct measurement. Therefore, to compensate the disturbances it is necessary to estimate them first. One possible structure for disturbance estimation is presented in Fig. 1a.

In this digital realization, extraction of the equivalent disturbance $q(k)$ is done by using the nominal plant model $G_n(z)$. Mismatch between the nominal model and the real plant inevitably exists due to parameter variations and unmodeled dynamics. Hence, the plant dynamics may be described as

$$G(z) = G_n(z)(1 + \delta G(z))\,, \qquad\qquad (4)$$

where perturbations are limited by the multiplicative bound of uncertainty $\left|\delta G(e^{j\omega T})\right| \leq \gamma(\omega),\ \omega \in \left[0, \pi/T\right]$. The extracted equivalent disturbance is then obtained in the form

$$q(k) = d(k) + G_n(z)\delta G(z) u_k(k)\,, \qquad\qquad (5)$$

indicating that it carries information about the external disturbance, parameter perturbations and unmodeled dynamics. Plant output, as a function of control and disturbance, may be expressed according to Fig. 1a as

$$Y(z) = \frac{G_n(z)(1+\delta G(z))}{1+G_k(z)G_n(z)\delta G(z)}U(z) + \frac{1-G_k(z)G_n(z)}{1+G_k(z)G_n(z)\delta G(z)}D(z) \qquad\qquad (6)$$

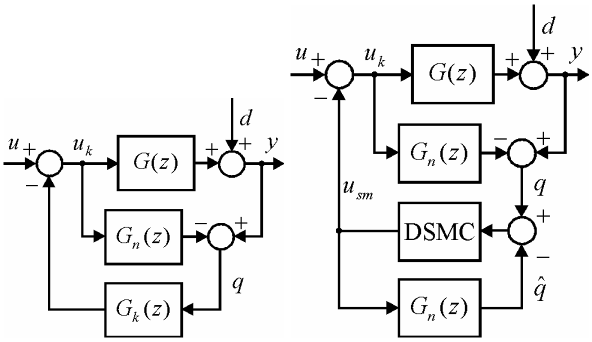

Fig.1. a) *Disturbance estimator*; b) *ADE based on DSMC.*

If compensation filter $G_k(z)$ represents nominal plant inverse dynamics, $G_k(z) = G_n^{-1}(z)$, the output becomes $U(z) = G_n(z)U(z)$. This shows that all disturbances are canceled and that the nominal plant behavior is ensured. Unfortunately, such filter is non-causal and cannot be realized.

Solution is proposed in [3] through the concept of ADE, Fig. 1b, where passive filter is replaced by an actively controlled subsystem. If DSM controller within ADE provides $\hat{q}(k)=q(k)$ by providing an ideal DSM, controller output becomes $U_{sm}(k)=G_n^{-1}(z)Q(z)$, showing that this subsystem acts as nominal plant inverse dynamics. Thus, complete disturbance rejection is achieved and nominal plant behavior is secured. This approach transforms the disturbance compensation problem into a tracking problem of the referent signal *q*(*k*). In the tracking subsystem of ADE, DSM controller governs nominal model, not the real plant, so there are no uncertainties and all state coordinates are available. Generally, due to the not known in advance referent signal *q*(*k*), it is possible to establish only quasi-sliding regime [14], resulting in nonideal disturbance rejection. However, since DSMC systems provide high-accuracy tracking, an excellent compensation, i.e. near nominal behavior is expected.

5. DSM CONTROLLER DESIGN

Let the nominal plant model is in the form (1) without the disturbance therm. Since control objective is to force the output *y*(*t*) to track the reference *r*(*t*), it is more convenient to transform the system model into canonical tracking error space, $e_1=r-y=r-x_1$, $e_2=\dot{e}_1=\dot{r}-x_2$, which is obtained as

$$\dot{\mathbf{e}}=\mathbf{A}\mathbf{e}-\mathbf{b}(u+b^{-1}v_r)\text{, where } v_r(t)=\ddot{r}(t)+a\dot{r}(t)\,, \tag{7}$$

where *r* denoted referent position signal.

The additional disturbance term $\mathbf{b}b^{-1}v_r(t)$ occurs due to the transformation, and appears because the reference signal varies in time. The referent signal is not analytically known, hence the total elimination of its derivatives through the control part generally is not possible. Note also that in the case of a step input $v_r(t)=0$. Discrete-time model of (1) for the given sampling period *T* is obtained in the form

$$\begin{aligned}&\mathbf{e}(k+1)=\mathbf{A}_d e(k)-\mathbf{b}_d(u(k)-b^{-1}v_r(k)),\\&\mathbf{A}_d=e^{\mathbf{A}T},\mathbf{b}_d=\int_0^T\mathbf{e}^{\mathbf{A}t}\mathrm{d}t\mathbf{b},\end{aligned}\tag{8}$$

To attain zero tracking error employing DSM strategy, it is necessary to establish a DSM along the sliding surface *s*(*k*)=0, defined by the switching function

$$s(k)=\mathbf{c}\mathbf{e}(k),\quad \mathbf{c}\in\Re^{1\times 2}. \tag{9}$$

If stable sliding dynamics is secured by appropriate selection of vector **c**, the state space origin will be reached, and ideal tracking will arise. The adopted control algorithm that provides desired motion will be briefly exposed hereafter.

DSM controller within ADE, based on the algorithm proposed in [15], governs the nominal plant model with the only priority to ensure as accurate tracking as possible in order to gain the precise nominal model inverse dynamics. Due to finite sensor resolution, measurement uncertainties enter the control system through the equivalent disturbance *q*, which is the reference for the DSMC subsystem in ADE, Fig. 1b. The evaluated equivalent disturbance according to the measurements is contaminated by the quantization noise, i.e. $q_m=q\pm\Delta\theta$, where $\Delta\theta$ is the maximal position measurement error $\Delta\theta=\pi/N_{eff}$. N_{eff} is the sensor effective resolution, which in case of incremental encoder with quadrature decoder becomes $N_{eff}=4\cdot N$, *N* being the physical encoder resolution. Consequently, the calculated tracking error vector $\mathbf{e}_m(k)$, where the reference derivative is formed by Euler differentiation method, also contains uncertainties. Taking into account only dominant terms, $\mathbf{e}_m(k)$ may be written as

$$\begin{aligned}&\mathbf{e}_m(k)=\mathbf{e}(k)\pm\Delta\mathbf{e},\ \Delta\mathbf{e}=\begin{bmatrix}\Delta\theta & 2\Delta\theta/T\end{bmatrix}^{\mathrm{T}},\\&\mathbf{e}_m(k)=\begin{bmatrix}q_m(k)-x_1(k)\\(q_m(k)-q_m(k-1))/T-x_2(k)\end{bmatrix}.\end{aligned}\tag{12}$$

The introduced noise is further amplified by the numerical differentiation of the referent signal needed for the DSMC. It was shown in [13] that under these circumstances, assuming $|\ddot{q}(k)|\le M_1,\forall k$ and $\mathbf{c}\mathbf{b}_d=-T$, only a quasi-sliding regime would be possible, even in nominal systems, in the domain

$$S_{qs}=\{\mathbf{e}\,\big|\,|s(\mathbf{e})|\le|\mathbf{c}\mathbf{A}_d\Delta\mathbf{e}|+(TM_1+2a\Delta\theta)/b\}\,. \tag{13}$$

The measurement uncertainties threaten to invoke chattering, so the applied DSMC algorithm within ADE should be less sensitive to the noises and measurement uncertainties. A new DSMC algorithm [13] is employed that avoids chattering by imposing a two-scale reaching law. The promoted idea is to slow down system trajectories near the sliding surface in order not to excite unmodeled dynamics. The proposed reaching law is given by

$$\begin{aligned}&s(k+1)=s(k)-\sigma(s)T\,\mathrm{sgn}(s(k)),\\&\sigma(s)=\begin{cases}\sigma_1, & |s(k)|>m\sigma_1T,\\ \sigma_1/n, & |s(k)|\le m\sigma_1T,\end{cases}\end{aligned}\tag{14}$$

where $\sigma_1,m,n>1,\ n>m$. Besides fast convergence zone (FCZ), system has slow convergence zone (SCZ) as well, where the convergence rate is *n* times smaller. The control is defined as

$$\begin{aligned}u_s(k)=&-T^{-1}\mathbf{c}(\mathbf{A}_d-\mathbf{I})\mathbf{e}(k)-\\&\sigma(s)\,\mathrm{sgn}(s(k))+ab^{-1}\dot{q}(k)\end{aligned},\tag{15}$$

where Euler's approximation is used for the reference derivative. The resulting quasi-sliding domain and the sufficient condition for the convergence into that domain are described respectively by the following inequalities

$$|s(k+1)|\le T[\sigma(s)+M_1/b],\quad \sigma(s)>M_1/b\,. \tag{16}$$

In order to adequately set the boundaries of the SCZ, as well as to obtain maximal accuracy determined by (13), parameters *m* and *n* should satisfy the following conditions

$$m>1+\frac{M_1}{b\sigma_1},\quad n>\frac{\sigma_1T}{|\mathbf{c}\mathbf{A}_d\Delta\mathbf{e}|+b^{-1}(TM_1+2a\Delta\theta)}\,. \tag{17}$$

5.1 Supplemental Integral Action

To secure convergence inside the SCZ in case of the perturbed systems and to increase system accuracy, DSM controller is further enhanced by an additional integral action with respect to the sliding variable *s*(*k*), [5]. Integration is run only in the vicinity of the sliding surfaces with a constant integral gain *h*. Stability is not jeopardized if 0 < *h* < 1/*T*. The introduced control component is given by

$$u_I(k)=\begin{cases} u_I(k-1), & |s(k)|> m\sigma_1 T,\\ hs(k)+u_I(k-1), & |s(k)|\le m\sigma_1 T.\end{cases} \tag{18}$$

Output of the enhanced DSM controller now becomes

$$u_{sm}(k)=u_s(k)-u_{Iadc}(k)\,. \tag{19}$$

Finally, the overall control signal, applied at the plant input, is composed of the outputs of the main PD and the DSM controller of ADE in the following way

$$u_k(k)=u_r(k)-u_{sm}(k)\,. \tag{20}$$

3.4 Sliding Surface Design

Sliding surface of the corresponding DSM controller is constructed according to the sliding dynamics defined by a desired eigenvalue $z_1=e^{-\alpha T}$, $\alpha>0$. For the given controllable pair $(\mathbf{A}_d,\mathbf{b}_d)$, vector **c** is defined by

$\mathbf{c}=T\begin{bmatrix}z_1 & -1\end{bmatrix}\mathbf{P}_1^{-1}$, where

$$\mathbf{P}_1=\begin{bmatrix}\mathbf{b}_d & \mathbf{A}_d\mathbf{b}_d\end{bmatrix}\begin{bmatrix}a_1 & 1\\ 1 & 0\end{bmatrix},\ \det(z\mathbf{I}-\mathbf{A}_d)=z^2+a_1z+a_0. \quad (2$$

6. EXPERIMENTAL RESULTS

The effectiveness of the proposed structure has been investigated by experiments on a servo system with a low power permanent magnet DC motor. The motor has been identified in the form of (1) with parameters *a*=26.38, *b*=654.35. Position is measured by an incremental encoder with the resolution of 0.00157 rad. Control part of the system has been implemented by dSPACE DS1104 R&D controller board, installed on a host computer. Sampling period is *T*=0.001s.

The main PD controller $G_r(z)=k_p+k_d(z-1)/z$ has been tuned by $k_p=0.603$ and $k_d=20.294$, providing relative damping factor $\zeta=1$ and undamped natural frequency $\omega_n=20$ in the nominal system. Sliding dynamics of the DSM controller inside ADE is set by $\alpha=200$ that gives **c**=[-0.2814 -0.0014]. The remaining controller parameters are *h*=30, $\sigma_1=2.5$, *m*=5 and *n*=4.

The system has been subjected to a step angle command *r*(*t*)=10*h*(*t*-0.1) rad under action of an external disturbance through the step load torque $T_l(t)=0.1h(t-1)\,\mathrm{Nm}$, which makes 50% of the nominal torque. Position responses is shown in Fig. 2 in case of the PD controller only (line (1)) and the proposed structure with combined action of the PD controller and ADE (line (2)). The conventional system gives poor results due to the controller impossibility to overcome considerable dry friction and to reject external disturbance. Activation of ADE drastically improves system performance. According to the system output that is presented in the enlarged scale in Fig. 3, it is evident that ADE completely compensates dry friction and load torque, and provides maximal possible accuracy within the sensor resolution. The corresponding control signals are given in Fig. 4. Switching function of the DSM controller that is incorporated inside ADE is presented in Fig. 5. It may be noticed that DSM controller ensures occurrence and existence of the quasi-sliding mode along the chosen surface.

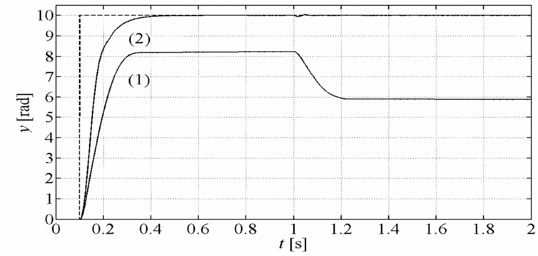

Fig. 2. *Position response: (1)-PD controller only; (2)-PD+ADE control structure.*

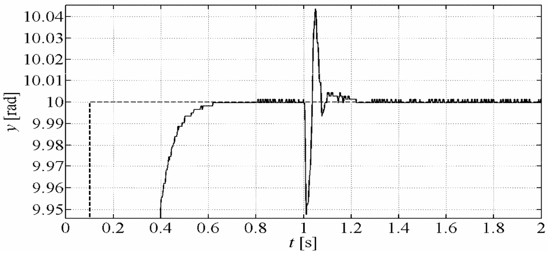

Fig. 3. *Position response of the PD+ADE structure (enlarged scale).*

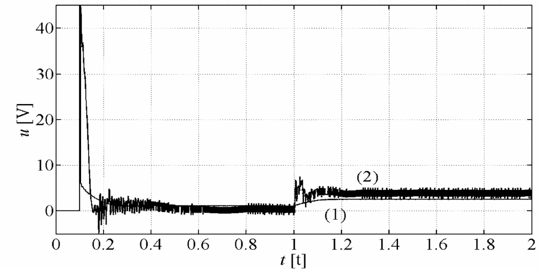

Fig. 4. *Control signals: (1)-PD controller only; (2)-PD+ADE control structure.*

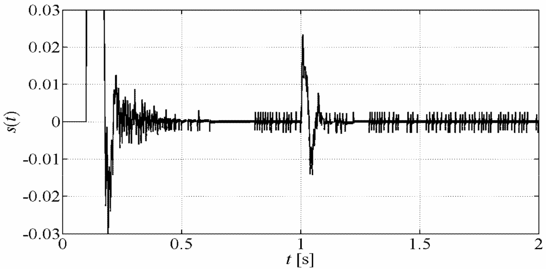

Fig. 5. *Switching function.*

The experiment has been repeated in case of sinusoidal load torque $T_l(t)=0.1\sin(10t)h(t-1)\,\mathrm{Nm}$. The corresponding results are shown in Fig. 6 to Fig. 9. According to the given plots it may be inferred that the designed system equally well eliminates nonlinear external disturbances. It confirms that the proposed control extension turns the initial conventional system, with limited possibilities, into an efficient positioning system.

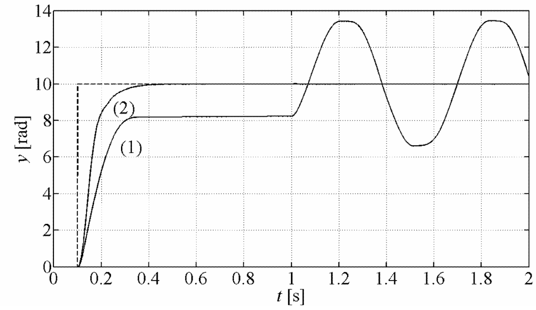

Fig. 6. *Position response: (1)-PD controller only; (2)-PD+ADE control structure.*

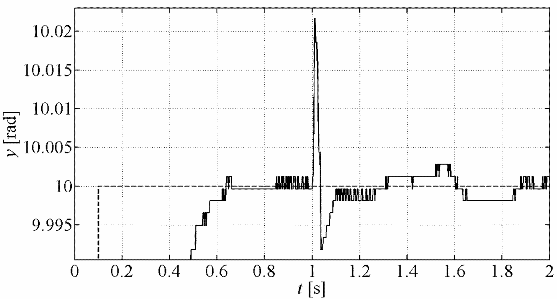

Fig. 7. *Position response of the PD+ADE structure (enlarged scale).*

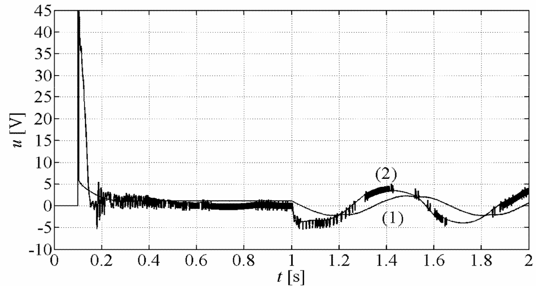

Fig. 8. *Control signals: (1)-PD controller only; (2)-PD+ADE control structure.*

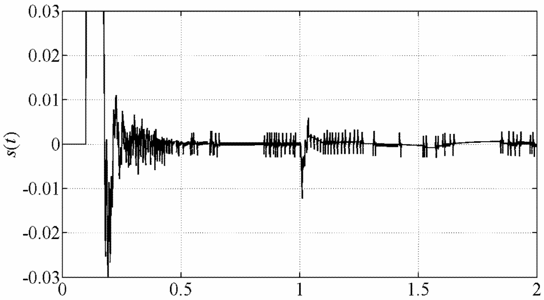

Fig. 9. Switching function.

7. CONCLUSIONS

The paper proposes a way to upgrade conventional servo-systems by introduction of digital ADE. Adjoining of SM based ADE with additional integral action and two scale reaching law improves system performances in position accuracy with effective disturbance rejection. ADE contains active DSM controlled subsystems, whose controller is designed for the nominal plant. Experimental results evidently show that the proposed control extension ensures superior performance comparing to the initial system, confirming that the conventional positional system turns into a robust high-performance positional system.

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