Paper No. T4-2.2, pp. 1-5

15th INTERNATIONAL SYMPOSIUM on POWER ELECTRONICS - Ee 2009
XV Međunarodni simpozijum Energetska elektronika – Ee 2009
NOVI SAD, REPUBLIC OF SERBIA, October 28th - 30th, 2009

CALCULATION OF BASIC ELECTRIC VALUES USING THE DEFINITION FORMULAS

Predrag B. Petrović
University of Kragujevac, Technical Faculty Čačak, Republic of Serbia

Abstract: *The paper proposes a practical new algorithm for calculation of basic ac (alternate current) electric values: average, rms, active power and energy. Based on the value of the integration of the original input (analogue) signal, a calculation is performed using the definition formulas. The realization avoids the use of a separate circuit for sample and hold, and the analogue signal was directly introduced in the integration circuit, after which its digital equivalent is formed. The suggested measuring system was simulated, realized in a practical set-up, and tested. The errors related to the signal processing and errors bound were investigated and provided.*

Key Words: ***AC electric values, time-domain, integration, definition formula***

1. INTRODUCTION

Many applications involve digital processing of periodic signals. For example, both voltage and current in electrical utilities are periodic signals containing harmonic components [1-4]. Often due to existing physical limitations, the available data is insufficient to achieve high resolution in processing, which is especially important for the problems that occur during the conversion of the observed signals, due to the non-ideal nature of the AD converter that is used here, as well as the transducer and the other devices used in the process [5]. On the other hand, sampling consists of a sample and hold (or track and hold) circuit followed by conversion to digital code word, and is implemented using an AD converter. A sample and hold circuit is a possible source of systematic errors. In [6], attention is paid to the possibility of eliminating a separate circuit for sample and hold, which in itself is an original approach. An important advantage of this method (integration) is that the input signal becomes averaged as it drives the integrator during the fixed-time portion of the cycle. Any changes in the analogue signal during that period of time have a cumulative effect on the digital output at the end of that cycle. Other ADC strategies merely "capture" the analogue signal level at a single point in time in every cycle. If the analogue signal is "noisy" (contains significant levels of spurious voltage spikes/dips, as it almost always happens in practice), one of the other ADC converter technologies may occasionally convert a spike or dip because it captures the signal repeatedly at a signal point time. The processing algorithm as suggested by this paper differs from the one explained in [6], inasmuch as it does not require an additional solving of a system of equations in order to establish the basic ac values, which makes it considerably less demanding from the point of view of practical realization.

The paper offers a new approach in processing analogue signals (most of all ac signals that can be represented in the form of Fourier series, such as ac signals of electric current and voltage), while the processing in question can also include non-harmonic elements (interharmonics and subharmonics). Based on the results presented in [7], we can adopt suggested algorithm for the accurate processing off all relevant quantities of such complex input ac signals. The suggested processing method pays special attention to the possible sources of error. The proposed procedure depends on the frequency f of the carrier signal, but the problem of synchronization is much less expressed than with the standard techniques of synchronized sampling [8]. The method suggested here does not depend on complex hardware or intensive numerical calculations, which makes its practical realization highly inexpensive.

2. SUGGESTED METHOD OF PROCESSING

Let us assume that the input signal of the fundamental frequency f is band limited to the first M harmonic components. This form of the continuous signal (voltage or current) with a complex harmonic content can be represented as a sum of the Fourier components as follows:

$$x(t) = X_I + \sum_{k=1}^{M} X_k \sin(k2\pi ft + \psi_k) \qquad (1)$$

Here X_I is the average value of the input signal; X_k is the amplitude of the *kth* harmonic; k is the number of the harmonic; and ψ_k is the phase angle of the *kth* harmonic. By integrating the signal (1) (similarly, to what we did in equation (7) [6]), through the integrator and the accompanying logic that can be found in a dual-slope or sigma-delta ADC (Figure 1), we obtain:

$$Y(t_1)=\frac{1}{RC}\int_{t_0}^{t_1}x(t)d\tau=\frac{1}{RC}\left\{\begin{matrix}X_I(t_1-t_0)+\\ +\frac{1}{\pi f}\sum_{k=1}^{M}\frac{X_k}{k}\sin(2k\pi f t_1+\psi_k)\sin\left[k\pi f(t_1-t_0)\right]\end{matrix}\right\} \qquad (2)$$

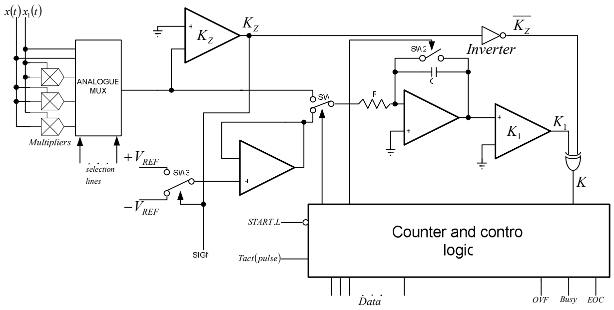

Fig.1. *Principal block-diagram of digital circuit for realization of proposed algorithm.*

The time interval t_1-t_0 is an interval of a constant length, within which the integration of the input analogue signal is done. The moment t_0 is defined as the initial moment, from which the integration process begins in the above relation is arbitrary. The $Y(t_1)$ value represents the value of the integral and is calculated based on a certain reference value used in the process of conversion, i.e. in the work of the circuit with which the integration is performed.

If we assume that the interval t_1-t_0 is equal to the period of the input signal $T=1/f$, then the operation described in equation (2) practically calculates the average value of the input analogue signal according to the definition formula, i.e.:

$$Y(t_1)=\frac{1}{RC}\cdot\overline{x(t)}\cdot T \qquad (3)$$

where $T=t_1-t_0=1/f=kt_c$, the time in which the counter registered k clock impulses of period t_c. Within this precisely defined interval, the counter managed to count off, up to the value of a certain content that is remembered inside the accompanying logic (the microprocessor that controls the operation of the complete system in Figure 1). The digital equivalent to the calculated value is established by the means of the counter and the reference voltage signal, following the same principle by which this is done inside a dual-slope or sigma-delta ADC. Namely, after the integrator performs the re-calculation according to the relation (2) or (3), in the second time interval, dependent on the speed of the counter (counter's tact signal) that is being used, a numerical equivalent of the calculated integral of the analogue input signal is determined, based on a known reference value. At the end of the second interval (from the moment t_1, to the moment t_2), the comparator placed at the integrator output will identify the moment t_2 as the moment in which the total voltage at the integrator output reached the 0 value, i.e.:

$$Y(t_2)=Y(t_1)-\frac{1}{RC}\int_{t_1}^{t_2}V_{REF}d\tau=0 \qquad (4)$$

where $T_2 = t_2 - t_1 = it_c$, the time in which the counter registered i clock impulses, V_{REF} is reference voltage. From the last formula it can be seen that the number i, that is, the number that was registered at the counter at the moment t_2, is proportional to the average value of the input signal and is not dependent on the resistance R, capacitance C, nor on the period of the clock impulses t_c. If further developed (4) this results in:

$$Y(t_2)=Y(t_1)-\frac{1}{RC}\int_{t_1}^{t_2}V_{REF}d\tau=\frac{1}{RC}\overline{x(t)}\cdot T-\frac{1}{RC}V_{REF}\cdot i\cdot t_c=0\Rightarrow \qquad (5)$$

$$\overline{x(t)}=\frac{V_{REF}\cdot i\cdot t_c}{T}=\frac{V_{REF}\cdot i\cdot t_c}{k\cdot t_c}=V_{REF}\frac{i}{k}$$

In order to perform precise processing according to the proposed concept, it is necessary to conduct the process of the integration of the input signal very precisely; this is to say that the definition of the interval t_1-t_0 must be precise (with this interval equal to the period of the processed signal). In the suggested algorithm the frequency of the carrier signal is recalculated in every passage, in a way that takes into consideration a possible change, introducing it in the process of recalculating the new value of the input analogue signals integral $Y(t_1)$. In this way, we will reduce the possibility of error in the calculation process that way appear as a result of the variation in the frequency of the processed signal. After the numeric equivalent of the integral was determined in the second interval, it will be necessary to divide the contents of the counter from the second passage with its contents from the first integration interval (which is determined by the input signal period); in this way we obtain the average value of the signal which is being processed (equation (5)). We should note that the sign of V_{REF}, is dependent upon the sign of $Y(t_1)$, that is to say, if $Y(t_1)$ is positive, then V_{REF} must be negative, and vice versa. The sign of the expression $Y(t_1)$ can be verified by a program check, thereby adjusting the sign of V_{REF}, which can be avoided if the operation is conducted using the absolute values of the operation.

In order to calculate the rms (root mean square) value of the input signal, it is necessary to multiply the signal by itself. This must be done before the integrator, in the way, it was done through the multiplier, which is located before the analogue multiplexer in Figure 1, after which we can perform the described processing procedure. In the end, what remains to be done is to calculate the square root of the calculated mean value of the signal's square. The power in the AC system is obtained when we multiply the values of the voltage and current signals, integrate them, and calculate the mean value of the signal obtained in this way.

By using the approach without a separate sample and hold circuit, the suggested algorithm is utterly different from the method described in [6], because it does not require an additional solving of a system of equations in order to establish the basic ac values, which makes it considerably less demanding from the point of view of the processor power and the time needed for the realization. It makes it possible to determine the digital equivalent of the desired electrical values (of the processed analogue signals) in the most direct way, based on the derived relations (3)-(5).

The time needed to reach the required value (average, rms, active power or energy) is in the border case, limited to the time interval that is marginally longer than the two periods of the input signal that is the subject of the processing. As a result of this, the processing algorithm described here is not limited only to slow-changing ac signals, unlike the methods proposed in [6] and [7].

The method suggested here does not depend on complex hardware or intensive numerical calculations, which makes its practical realization highly inexpensive. Also, it is clear that this kind of approach puts no limitations whatsoever on the speed of the circuit through which the conversion is done, i.e. the determination of the numerical value of the continual signal integral, in the form of the Fourier series. This is an important difference to be taken into consideration when comparing its implementation in this approach to the processing, to the results presented in [6], that was based only on the use of standard dual-slope ADC. To realize the circuit with which the integration is done, it is necessary to use a separate circuit of the integrator with the addition of a resetting option, to annihilate the impact that the time constant has upon the precision in the process of integrating the value of the input analogue signal. After this, the counter will form a digital equivalent to an integral obtained in this way. Based on the proposed solution shown in figure 1, it can be assumed that the circuit can be realized in a very simple manner, as an IC.

This kind of processing does not introduce limitations regarding the type of the signal that is being processed, i.e. this signal can contain non-harmonic components as well (interharmonics and subharmonics); however, it will be necessary to determine the period of such a complex signal, as it was done in [8].

2.1. Estimation of measuring uncertainty

The measuring uncertainty of the realized digital multimeter is a key metrological characteristic. In order to achieve a very low measuring uncertainty, all of the decision making in producing the concept for, as well as in the development and realization of this instrument, was restricted by extreme requirements. A model of the results of measuring can be formulated based on expression (5). This formulation includes the factors that can have a significant impact on the measuring uncertainty of the used method:

$$\overline{x(t)} = \left(V_{REF} + \Delta V_{REF}\right)\frac{\left(i + \Delta i + \delta i\right)}{\left(k + \Delta k + \delta k\right)} \qquad (6)$$

- ΔV_{REF} - correction of the reference voltage. This is a random value, with an even distribution. Its mathematical expectation is equal to zero, and its relative standard measuring uncertainty $u_{\Delta REF}$ of the B type, is determined by the manufacturer's specification.
- Δi - correction of the i - counter content. This value depends on the precision of the operation of the zero comparator. If the procedure of eliminating the error of the comparator is applied (the so-called 'auto-zeroing'), this correction will be neglected, together with its uncertainty.
- δi - inherent error of the counter: ±1. This is a random value, with an even distribution. Its mathematical expectation is equal to zero, and its standard measuring uncertainty is $u_{\delta i} = 1/\sqrt{3}$.
- Δk - correction of the k - counter content. This value depends on the precision in the measuring of the period of the processed signal, $\Delta k = \frac{\Delta T}{t_c}$. ΔT is the difference between the period of the measured and the processed (real) signal. Δk is a random value, with an even distribution. Its mathematical expectation is equal to zero, and its relative standard measuring uncertainty $u_{\Delta k}$ of the B type, is determined as $u_{\Delta k} = \frac{u_{\Delta T}}{t_c}$, where $u_{\Delta T}$ represents the measuring uncertainty in the evaluation of the period of the processed signal.
- δk - inherent error of the counter: ±1. This is a random value, with an even distribution. Its mathematical expectation is equal to zero, and its relative standard measuring uncertainty is $u_{\delta k} = 1/\sqrt{3}$.

A combined measuring uncertainty is calculated according to the well-known rules [9]:

$$u_x^2 = \left(\frac{i}{k}\right)^2 u_{\Delta REF}^2 + \left(\frac{V_{REF}}{k}\right)^2 u_{\Delta i}^2 + \left(\frac{V_{REF}}{k}\right)^2 u_{\delta i}^2 + \left(-\frac{V_{REF} i}{k^2}\right)^2 u_{\Delta k}^2 + \left(-\frac{V_{REF} i}{k^2}\right)^2 u_{\delta k}^2 \qquad (7)$$

Example: For an effective value of the processed signal that is close to the reference value, and a 16-bit counter that operates at tact of 10MHz, together with an error in determining the frequency of the processed signal that is within the ±0.1Hz, the standard measuring uncertainty of the method will be approximately 0.35%. An analysis of the measuring uncertainty reveals that a dominant influence of the uncertainty occurs as a result of the evaluation of the frequency of the processed signal.

3. SIMULATION OF THE SUGGESTED MEASURING METHOD

Additional testing of the proposed algorithm was carried out by simulation in the program package Matlab (version 7.0).

Since measurement is corrupted by noise, the processing is an estimation task, depending on the actual noise record. The estimated input signal of the measurement system consists of two types of errors: systematic and stochastic ones. In the proposed system for signal processing, we eliminated a sample-and-hold circuit as a possible source of systematic errors. The impact of noise and jitter on the measurement method is investigated. In simulations, after a complex harmonic input signal was formed, the white Gauss noise was added to the measuring signals, having defined the amplitude range. In this way, it is possible to get a general insight into the impact that the noise and the harmonic components have on the precision of the suggested algorithm.

The presence of the noise and jitter causes false detection of signal zero-crossing moments giving an incorrect calculation of the interval for integration of the input signals (equation (3)). The error analysis was performed both in the program and as a SIMULINK-u (Figure 2), in order to verify the effectiveness of the proposed hardware solution (Figure 1).

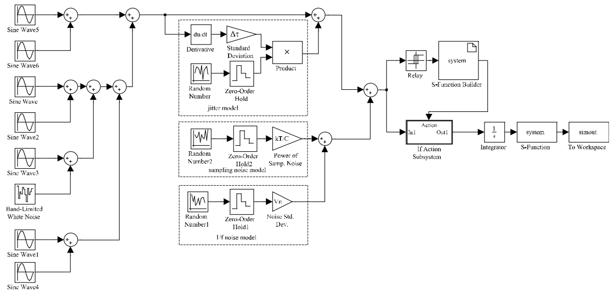

Fig. 2. *SIMULINK model of proposed hardware realization.*

Figure 2 shows the addition of the white Gauss noise, thermal and 1/*f* noise, and jitter to the complex periodic signal. The power of the noise in the digital circuit suggested for the realization of the proposed algorithm (figure 1) was well-described and defined in [10]. Considering that the noise power is additive, the PSD (Power Spectral Density can be considered as the sum of a term due to flicker (1/*f*) noise and one due to thermal noise, associated to the sampling switches and the instrinsic noise of the operation amplifier. However, in order to have the most precise analysis of the sampling noise and the op-amp's thermal and flicker noise at low frequencies, these were modeled by separate blocks in Figure 2. Here, *k*, *T*, and C are Boltzmann's constant, the temperature in Kelvin, and the sampling capacitor, respectively. The V_n denotes the input-referred thermal noise of the op-amp. Flicker (1/*f*) noise, wide-band thermal noise and dc offset contribute to this value. The total noise power V^2_n can be evaluated, through transistor level simulation, and during the simulation it is assumed that V_n=30 μV_{rms}, while the value of the sampling capacitance was C=2 pF. The effect of jitter was simulated by using a model separately shown in Figure 2, under the assumption that the time jitter is an uncorrelated Gaussian random process having a standard deviation $\Delta\tau$. This model of the jitter was described and analyzed in detail in [10], where it was shown that the noise floor is dependent on the input sinusoidal frequency. During the simulation, the value of jitter was 1ns (standard deviation $\Delta\tau$). In the course of the simulation conducted in this way, the signal-to-noise distortion ratio (SNDR) ranged between 56 dB and 96 dB.

By using the simple Relay block shown in Figure 2, the frequency of the fundamental harmonic was measured in the signal formed in this way. This block produced a series of samples to be used as input data for „S-Function Builder". Inside it, and based on the proposed algorithm the integration interval is defined, which practically generated the signal action (enable signal) in the block „If Action Subsystem". This action determines the time interval in which integration of the input signal was done. If the moments of the zero crossing of the signal that is processed are precisely determined, the accuracy of period reading then depends only on the processing power of the realized measuring system. Figure 3 show the influence of the error in determining the frequency of the carrier signal on the relative error in determining the rms value of the input signal, for various harmonic content of the input periodic signal.

According to the results of the analysis shown in Figure 3, it can be concluded that there is a practical linear dependence between the size of the relative error in determining the rms value of the input signal on one side, and the error in determining the frequency of the fundamental harmonic, on the other side. This error becomes bigger when establishing the rms value than in case of calculation of the average power – this actually happens as a result the need to calculate the square root of a certain integral. However, the limits of the possible errors are acceptable; since the allowed tolerance in the network frequency is rather low, (the first security level in the network is activated when there is a change in frequency of 0.05Hz in relation to the nominal value). The immunity of the algorithm could be improved by applying a more complex algorithm for the detection of signal zero-crossing moments [8].

This kind of processing does not introduce limitations regarding the type of the signal that is being processed, i.e. this signal can contain non-harmonic components as well (interharmonics and subharmonics); however, it will be necessary to determine the period of such a complex signal, as it was done in [7]. It is absolutely clear that proposed approach puts no limitations whatsoever on the speed of the circuit through which the conversion is done, i.e. the determination of the numerical value of the continual signal integral, in the form of the Fourier series. A new algorithm practically enables the processing to be performed on-line, thus enabling high resolution and speed in processing of input analogue signals. Based on the proposed solution shown in Figure 1, it can be assumed that the circuit can be realized in a very simple manner, as an IC circuit.

The proposed digital instrument was simulated in the laboratory. In order to test the accuracy of the proposed method of measurement, parallel measurements of rms voltage and current, and active power with an extremely precise multimeter, Hewlett Packard HP 3458 A were carried out. The obtained results completely proved our assumptions (Table 1). The results show that the accuracy of the proposed algorithm is acceptable under all conditions. It is independent of the harmonic content of the voltage and current signals and has high accuracy even if the frequency deviates from the nominal value. The very simple and inexpensive hardware, in contrast to the highly sophisticated and expensive hardware described in [10], meets all price and accuracy requirements for the design of the measurement system. The high-level accuracy in processing ac values is preserved, better then in some other solutions [11], with excellent noise rejection.

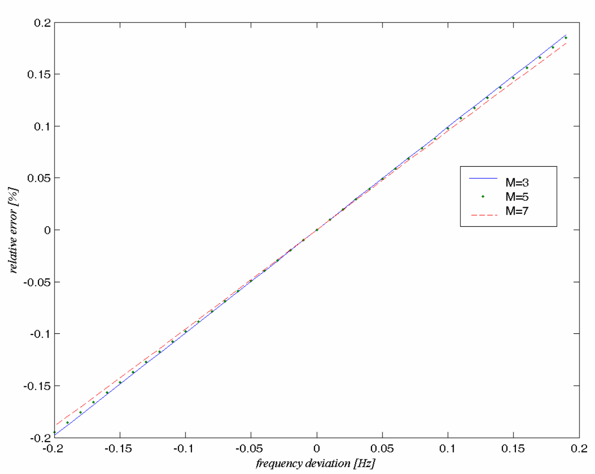

Fig. 3. *Relative error in calculation of input signal rms value as function of error in synchronization with frequency of fundamental harmonic of the input signal (M is the number of the highest harmonic component).*

Table 1. *The results of experimental measuring with the proposed digital instrument.*

Rms values of voltage and current of the source; the phase between voltage and current signals	Measured rms value of voltage (relative errors [%])	Measured rms value of current (relative errors [%])	Measured value of active power (relative errors [%])
120V; 6A; 0°	0.00089	0.00095	0.00062
120V; 5A; 30°	0.00083	0.00085	0.00065
120V; 4A; 60°	0.00078	0.00088	0.00054
110V; 6A; 30°	0.00085	0.00092	0.00063
110V; 3A; 0°	0.00068	0.00075	0.00051
110V; 1A; 45°	0.00092	0.00096	0.00071
100V; 4A; 60°	0.00084	0.00091	0.00072
100V; 2A; 0°	0.00089	0.00093	0.00065
60V; 2A; 30°	0.00092	0.00096	0.00073
60V; 1A; 60°	0.00091	0.00097	0.00075

4. CONCLUSION

Through the given expression, which is practically an on-line one for the known fundamental frequency of input signals, we determine all of the unknown basic electric values. Based on the suggested concept of processing, the complete realization of the necessary hardware can be achieved through the resources of the microprocessor itself (only the most modern type of processor is satisfactory here, however, with a considerably high performance), as well as through its counting resources. All these elements will lead to reduced costs of the suggested solution. What has been avoided here is the use of a separate sample-and-hold circuit which, as such, can be source of a system error. The main advantage of the digital multimeter described in this paper is that the measurements are performed exactly according to the determination of power and true rms voltage. The necessary synchronization is achieved by software measurements of the frequency of the measured signal. The analysis shows that the proposed algorithm remains highly accurate in processing of periodic signals in real environment. The real limit of precision was found experimentally, and it was 0.001%.

References

[1] J. Xi and J.F. Chicharo, “A new algorithm for improving the accuracy of periodic signal analysis”, *IEEE Trans. Instrum. Meas.*, vol.45, no.4, pp.827-831, 1996.

[2] R. Rausher and V. Grupe, “An A/D-chip for accurate power measurement”, *Euro ASIC 1991*, pp. 352-355, May 1991.

[3] M. Dupplis and C. Svensson, “Low power mixed analog-digital signal processing”, *Proc. 2000 Inter. Sym. Low Pow. Elec. Des., ISLPED 2000*, pp.61-66, 2000.

[4] E. M. Hawrysh and G. W. Roberts, “An integration of memory-based analog signal generation into current DFT architectures”, *IEEE Trans. Instrum. Meas.*, vol. 47, no. 3, pp.748-759, 1998.

[5] J. G. Proakis and D. G. Manolakis, *Digital Signal Processing: Principles, Algorithms, Applications*, 3rd ed. Englewood Cliffs, NJ: Prentice-Hall, 1996.

[6] P. Petrovic, S. Marjanovic, and M. Stevanovic, „Measuring of slowly changing AC signals without sample and hold circuit“, *IEEE Trans. Instrum. Meas.*, vol. 49, no. 6, pp.1245-1248, 2000.

[7] P. Petrovic and M. Stevanovic,”Measuring of active power of synchronously sampled AC signals in presence of interharmnoics and subharmonics”, *IEE Proc., Elec. Pow. Appl.* 2006,vol. 153, no.2, pp.227-235, 2006.

[8] P. Petrovic, “New Digital Multimeter for Accurate Measurement of Synchronously Sampled AC Signals”, *IEEE Trans. Instrum. Meas.*, vol.53, no.3, pp.716-725, 2004.

[9] Z. Mitrovic, I. Zupunski: “Stable Source of AC Voltage and Current”, *Proceedings of IMTC 2004,* Como, Italy, May 2004.

[10] H. Germer, “High-precision ac measurements using the Monte Carlo method”, *IEEE Trans. Instrum. Meas.*, vol.50, pp. 457-460, 2001.

[11] http://us.fluke.com/usen/products/