



ROTOR RESISTANCE VARIATION INFLUENCE ON IFOC DRIVE OF INDUCTION MOTOR

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Abstract: This paper describes the induction motor Indirect Field Oriented Control (IFOC) when the IFOC model parameters do not match the actual motor parameters. The analysis has been made for a particular traction motor with nonlinearities taken into consideration. Based on the experimentally measured magnetizing curve, the influence of the rotor resistance mismatch on the electromagnetic torque, stator current components and rotor flux, is analyzed. The simulation results are verified experimentally.

Key Words: Induction motor/IFOC/Parameter variation/Nonlinear analysis/Rotor resistance

1. INTRODUCTION

In contemporary high performance AC motor drives, the most widely used scheme is an indirect field oriented control (IFOC) with current controllers in d - q reference frame tied to the rotor flux vector. In this type of drives, a mismatch between machine parameters used in the controller and the actual machine parameters, which occurs due to changes in temperature or saturation, results in the following [1]:

- The flux level is not properly maintained, and the rotor flux amplitude is not equal to the expected value.
- The resulting steady state torque is not equal to the the commanded values.
- The torque response is not instantaneous.
- The correlation between i_{qs} and torque, as well as the correlation between i_{ds} and flux is not linear anymore.

These effects are widely documented in literature [2,3]. In indirect field oriented control schemes, the parameters of greatest interest are rotor resistance and mutual inductance.

The wrongly set parameter in the model yields the difference between the commanded and obtained values of the currents even if the current controllers are ideal. Both, torque-producing (i_{qs}) and flux-producing (i_{ds}) currents are affected.

This paper discusses the relation between the expected values of the torque, flux and stator current components and the actual values, reached in the detuned drive. In the analysis, the nonlinearities of the main magnetic circuit have been taken into account, and the model considers the detuning.

1. MATHEMATICAL MODEL

A model of an induction machine with linear magnetics in the d - q frame (oriented so that the d -axis is aligned with the total rotor flux) is described by equations (1) – (5):

$$v_{ds} = R_s i_{ds} + \frac{d\psi_{ds}}{dt} - \omega_e \psi_{qs} \quad (1)$$

$$v_{qs} = R_s i_{qs} + \frac{d\psi_{qs}}{dt} + \omega_e \psi_{ds} \quad (2)$$

$$0 = R_r i_{dr} + \frac{d\psi_{dr}}{dt} \quad (3)$$

$$0 = R_r i_{qr} + (\omega_e - \omega_r) \psi_{dr} \quad (4)$$

$$T_e = \frac{3}{2} \frac{p}{L_r} M (\psi_{dr} i_{qs}) \quad (5)$$

In the above equations, subscripts d and q stand for the components in the d - q frame; and subscripts s and r denote stator, and rotor values, respectively. The other symbols are common, i for current, v for voltage, ψ for flux. R is resistance, L_r is total rotor inductance and M is mutual inductance. ω_e and ω_r are excitation (stator) and rotor angular frequency and p is the number of motor poles.

For current fed inverters, only equations (3) – (5) are of interest, assuming that the current controllers are much faster than the flux and mechanical dynamics. From the above equations and from the relations for the machine flux linkages, one can obtain [1]:

$$\omega_s = \omega_e - \omega_r = \frac{R_r}{L_r} \frac{M}{\psi_{dr}} i_{qs} \quad (6)$$

and,

$$R_r \psi_{dr} + L_r \frac{d\psi_{dr}}{dt} = R_r M i_{ds} \quad (7)$$

where ω_s is the slip angular frequency.

The last two equations define ψ_{dr} and as a function of stator currents, leading to the commonly used linear block diagram of an induction motor dynamics. The mathematical model described by this diagram allows the estimation of the instantaneous rotor flux position by adding the integral of ω_s (derived from the model using equation 6) to the measured rotor position [1].

3. DETUNED IFOC

In indirect vector control the torque and rotor flux amplitude are controlled by the references for d and q components of the stator current. The torque producing current reference i_{qs}^* (q -component) controls the torque while the d -component reference (or flux-producing current) i_{ds}^* controls the amplitude of the rotor flux. By means of i_{qs}^* the controller calculates the angular slip frequency as:

$$\omega_s^* = \frac{i_{qs}^* M^*}{\tau_r \Psi_{dr}^*} \quad (8)$$

where τ_r^* presents the rotor time constant value used as a parameter in the controller. The rotor time constant is defined as $\tau_r = L_r/R_r$. Stars in superscript for motor parameters identify the values used in mathematical model (in the controller) and are not necessarily equal to actual parameters in the machine.

The integral of ω_s^* , calculated using (8) presents the ‘‘slip contribution’’ to the rotor flux position, and is used to estimate (by adding to measured rotor position) current rotor flux position (θ_e^*), essential for coordinate transformations. Since the controller calculates ω_s^* using the wrong parameters, the estimated angle θ_e^* is not equal to the real rotor flux position (angle θ_e) in the machine. Unfortunately, the controller uses this, wrongly estimated, angle θ_e^* to rotate (transform from the stationary to rotated frame) measured stator currents $i_{\alpha s}$ and $i_{\beta s}$. The transformation (rotation) is necessary to get the d and q component of the stator current because the current controllers work in the synchronous rotating d - q frame.

The coordinate transformation, using the wrong angle, outputs the wrong value of the ‘‘measured’’ i_{d^*s} and i_{q^*s} (got after the rotation of $i_{\alpha s}$ and $i_{\beta s}$). The current controllers, even if ideal (with neither phase nor amplitude error) cannot ensure the equality of reference i_{qs}^* and i_{ds}^* currents with the torque-producing (i_{qs}) and flux-producing currents in the machine.

Instead, the current controllers equalize the reference currents to i_{d^*s} and i_{q^*s} , which are the only information about current components in d - q frame (obtained by the rotation of $i_{\alpha s}$ and $i_{\beta s}$, using incorrectly estimated θ_e^*).

On the other hand, somewhere in the machine, a real, well-oriented d - q frame certainly exists, but unfortunately, totally inaccessible for any kind of measurement. In this unreachable frame all presumptions connected to d - q frame are valid (flux in q -axes is 0, axes are independent, torque response is instant...). If we could manage to express the imaginary current components that would exist if the drive were tuned (i_{ds} and i_{qs}) as a function of the reference values (i_{ds}^* and i_{qs}^*) and some measure of the ‘‘mistune’’ of R_r , we could use a simple d - q model even for the detuned drive.

The expressions that connect reference currents and fictive d - q current components in inaccessible, correctly oriented d - q frame can be derived from two equations:

- The stator current amplitude is unique in any frame (in α - β frame, in correct d - q frame and in wrong, disoriented, d - q frame) [2].
- In a steady state, the angular slip speed in the correct, fictive d - q frame is the same as the value set in the detuned drive. If it was not, the error in rotor flux an-

gle, which is the integral of the difference between these two speeds, would never stop changing. Only if the difference equals zero, the integral can get a constant value. The strict proof for this can be found in [4].

The expressions for the stator current components in a correct, but inaccessible d - q frame that exists somewhere in the machine, developed using previous two conclusions are [4], [2]:

$$\begin{aligned} I_{ds} &= I_{ds}^* \frac{\sqrt{1 + (\omega_s \tau_r^*)^2}}{\sqrt{1 + (\omega_s \tau_r)^2}} \\ I_{qs} &= I_{qs}^* \frac{\tau_r}{\tau_r^*} \frac{\sqrt{1 + (\omega_s \tau_r^*)^2}}{\sqrt{1 + (\omega_s \tau_r)^2}} \end{aligned} \quad (9)$$

where capital letters emphasize that the expressions are valid in a steady state. Actually, they are valid in a quasi-steady state as well. It is enough to keep I_{ds}^* constant [4].

In the above equation, τ_r and τ_r^* are rotor time constants in the machine and in the controller, respectively, and the ratio τ_r/τ_r^* is the measure of the mistuning.

3.1. Detuned IFOC, nonlinear analysis

For a correct analysis of the detuned drive performance, the saturation of the main magnetic circuit must be taken into account. The reason for this is that the real flux-producing current (I_{ds}) in the machine may be even two times greater than the reference (I_{ds}^*) that pushes the machine deep into saturation. According to (9), for higher ω_s , if τ_r/τ_r^* is 0.5 (which might be expected due to the rotor heating), I_{ds} reaches $2 I_{ds}^*$. Figure 2 shows that the mutual inductance M , under these circumstances, is almost half of the expected value. Such a large variation in parameter values simply cannot be neglected.

A simple modified standard IFOC model with tabulated magnetizing curve [5] can take into account the changing of M with I_{ds}^* . In this model, M is not a constant but is being dynamically adjusted as $M(I_{ds}^*)$.

However, the modified model cannot compensate the variation of M completely. The mutual inductance in the machine is $M(I_{ds})$ while the best guess in the model can be $M(I_{ds}^*)$.

If we define the *impact of the saturation* as

$$\chi_s \equiv \frac{M(I_{ds})}{M(I_{ds}^*)} \quad (10)$$

and the rotor resistance error $\varepsilon \equiv R_r^*/R_r$, the previously defined measure of the detuning, τ_r/τ_r^* should be observed with more details when the saturation is considered. In the linear model, $\tau_r/\tau_r^* = R_r^*/R_r = \varepsilon$, but if M changes with I_{ds} , then,

$$\frac{\tau_r}{\tau_r^*} = \frac{L_r R_r^*}{L_r^* R_r} = \frac{M(I_{ds}) + L_{\sigma r} R_r^*}{M(I_{ds}^*) + L_{\sigma r} R_r} \approx \chi_s \varepsilon \quad (11)$$

with a presumption that the rotor leakage inductance $L_{\sigma r}$ is a constant and much lower than M .

Now, introducing the *load factor* defined as:

$$a^* = \frac{i_q^*}{i_d^*} = \omega_s^* \tau_r^* = \omega_s \tau_r \quad (12)$$

The expressions (9) can be rewritten as

$$I_{ds} = I_{ds}^* \frac{\sqrt{1+a^{*2}}}{\sqrt{1+\varepsilon^2 \chi_s^2 a^{*2}}} = I_{ds}^* \chi_d \quad (13)$$

$$I_{qs} = I_{qs}^* \varepsilon \chi_s \frac{\sqrt{1+a^{*2}}}{\sqrt{1+\varepsilon^2 \chi_s^2 a^{*2}}} = I_{qs}^* \varepsilon \chi_s \chi_d \quad (14)$$

with the *detuning impact factor* χ_d , defined as:

$$\chi_d \equiv \frac{\sqrt{1+a^{*2}}}{\sqrt{1+\varepsilon^2 \chi_s^2 a^{*2}}} \quad (15)$$

In expressions (13) and (14) the stator current components are defined as a function of an angular slip speed (hidden in the load factor a), for a given rotor resistance setting error ε , and for a known shape of magnetizing curve (which defines χ_s). All other parameters are constant in the controller.

Graphical presentation of these expressions for the experimental motor is given in figure 5. The curves had to be calculated numerically in several iterations. Namely, the saturation factor χ_s , in equation (13) is a

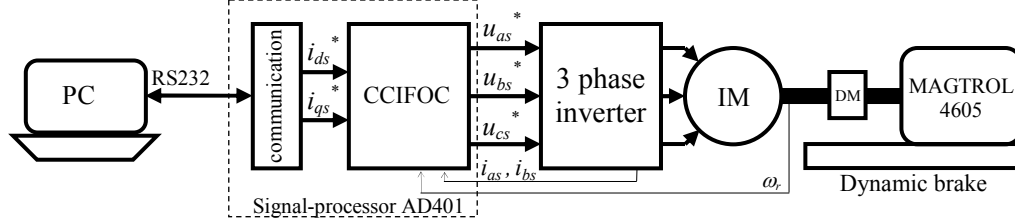


Figure 1 Experimental setup. PC provides the reference values for the torque that motor should produce and for the rotor flux level. The dynamometer DM measures the produced torque

function of I_{ds} itself. For the first iteration χ_s is set to 1 for all slip frequencies. The first iteration gave the set of I_{ds} values which were used to calculate χ_s for the next iteration. The iterations are repeated until the least square error between two consecutive iterations drops below 1%.

From equations (13) and (14) it is easy to derive the expressions for basic variables as a function of expected values and the rotor resistor setting error. For example, the amplitude of the rotor flux is $\psi_r = \psi_{dr} = M I_{ds}$,

$$\psi_r = M(I_{ds}) I_{ds}^* \frac{\sqrt{1+a^{*2}}}{\sqrt{1+\varepsilon^2 \chi_s^2 a^{*2}}} = M(I_{ds}) I_{ds}^* \chi_d \quad (16)$$

The equation (16) stresses that the mutual inductance is a function of a real I_{ds} , or, $M \equiv M(I_{ds})$. Since the flux of the tuned drive is $\psi_{r0} = M(I_{ds}^*) I_{ds}^*$, the equation (16) can take the form:

$$\psi_r = \frac{M(I_{ds})}{M(I_{ds}^*)} \chi_d \psi_{r0} = \chi_s \chi_d \psi_{r0} \quad (17)$$

Produced torque is:

$$T_e = \frac{3}{2} \frac{p}{2} \frac{(M(I_{ds}))^2}{L_r} I_{ds}^* I_{qs}^* \varepsilon \chi_s \frac{1+a^{*2}}{1+\varepsilon^2 \chi_s^2 a^{*2}} \quad (18)$$

or:

$$T_e = \chi_s^3 \chi_d^2 \varepsilon T_{e0} \quad (19)$$

Calculated values of the real torque and the real rotor flux as a function of the expected ones, and the angular slip frequency, are presented in Figure 6 for two values of ε and for two values of I_{ds}^* .

4. EXPERIMENTAL VERIFICATION

The aim of the experiment was to verify the expressions derived in chapter 3, that show the real value of the main variables (currents, fluxes, torque) in the detuned drive, as a function of the reference (or expected) value and the level of the mismatch between the real R_r and R_r^* in the controller. Due to the possibility of the produced torque measurement, all parameters of the particular machine are measured before the experiment, including the magnetizing curve.

The experimental results are obtained using an experimental setup based on a standard four-pole, 0.75kW induction motor. The motor is fed from a three phase inverter with an indirect vector controller and current controllers of i_{ds} i i_{qs} in a synchronous d - q frame tied to the rotor flux vector. CCIFOC is implemented on a signal-processor AD401 and it is connected to a PC using RS232.

Using the PC, the user controls two variables. One is the electro-magnetic torque that the motor should pro-

duce on the load (dynamic brake), and the other is the rotor flux amplitude (magnetizing of the motor). The two variables are controlled independently of each other and are limited only by the current capacity of the inverter and by the maximum motor current.

The torque is controlled by the q -component stator current reference value i_{qs}^* , while the d -component stator current reference value i_{ds}^* controls the rotor flux amplitude.

For the particular experimental motor, all parameter values, including the magnetizing curve (rotor flux as a function of I_{ds} in a steady state) are measured using standard IEEE112 [6]. Magnetizing curve is verified for the continually supplied machine (with sinusoidal signals) as well as for the PWM fed machine. A total of 25 points are measured and post-processed (off-line) with a local 5-point Savitzky-Golay smoothing filter [7]. For the first three points, a linear correlation is presumed. Between these points, a linear interpolation is made (new points are added) until the step of 0.1A for I_{ds} current. Thus, we ended with a 128-point table (rotor flux as a function of I_{ds}), used in the controller.

A dependence of the mutual inductance (M) on the magnetizing current (I_{ds} current, in a steady state) is then calculated and tabulated in the controller.

Mathematical model used in the controller [5] considers variations of the mutual inductance (M) with the current i_{ds}^* . The value of M used in the model dynamically tracks the changes of i_{ds}^* and present system state.

Figure 2 presents the magnetizing curve for the experimental motor and the relation between the mutual inductance (M) and the instantaneous value of I_{ds} (in a steady state, this current component is equal to the mag-

netizing current which defines the magnetizing curve). Both curves are in relative units. The base value for M is M_0 (mutual inductance in the linear part), and for the rotor flux ψ_r , as the base, its nominal value $\psi_{r,n}$ is used.

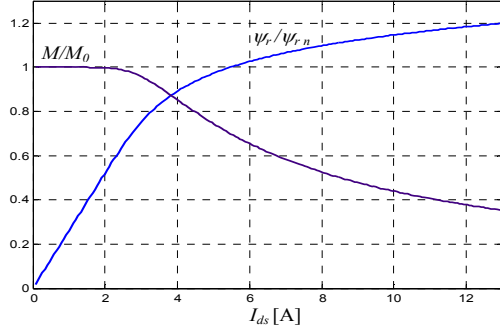


Figure 2 Experimentally obtained magnetizing curve $\psi_r/\psi_{r,n}$ and a relative value of the mutual inductance M/M_0 for the experimental motor. $M_0 = 163.7\text{mH}$, and $\psi_{r,n}$ is nominal rotor flux (0,59Wb)

Nominal torque of the motor is 4.15 Nm and is produced at the nominal slip speed $\omega_{snom} = 8 \text{ rad/s}$ ($f_{snom} = 1.27 \text{ Hz}$). Other parameters are as follows: stator resistance $R_s = 3.35\Omega$; rotor resistance $R_r = 1.99\Omega$; stator and rotor inductances $L_s = L_r = 170.7\text{mH}$ and the linear part mutual inductance $M_0 = 163.7 \text{ mH}$. Rotor time constant is $\tau_r = 85.7\text{ms}$.

Motor is mechanically connected to a dynamic brake Magtrol 4605 [8] which is set to the maximum available torque (24 Nm). As the motor cannot produce 24 Nm, the brake practically blocks the shaft, making a standstill conditions ($\omega_r = 0$) and in the experiment, only its dynamometer (Magtrol 4613 [8]) is used.

4.1 Influence of the R_r variation on the produced torque

Two series of measurements are made, both in a steady state.

- In the first series the nominal value of the flux-producing current ($I_{ds}^* = I_{dsnom} = 3,6\text{A}$) is used (Table 1).
- In the second, the reference value of I_{ds}^* was 1.8A (the half of the nominal).

Each series consists of three groups of 12 measurements.

- In the first group, the rotor resistance parameter in the controller R_r^* is set to the known, previously measured R_r (the real value in the machine). Hence, the controller is well tuned, the magnetizing of the machine is nominal and for each slip frequency, the machine should make expected torque on the load. The produced torque is measured for twelve slip frequencies, from 0.25Hz up to 3Hz, with the 0,25Hz step. Table 1, second column, marked with T_{e0} , presents the results of this measurement group.
- For the second group, the R_r^* parameter in the controller is set (on purpose) to a wrong value, equal to $R_r/2$, making the error $\varepsilon = 0.5$. The controller is detuned (by means of changed software), and the same twelve measurements are made. Table 1, third column, denoted with T_e , $\varepsilon = 0.5$, shows the values of the measured torque under these conditions.
- In the third group, the controller is put out of tune again, but in the opposite direction. This time R_r^* is set to $1.5R_r$, making the error $\varepsilon = 1.5$. The same twelve measurements are done, as in first two

groups. The results are in Table1, forth column, referred to as T_e , $\varepsilon = 1.5$.

The slip frequency f_s is not a variable under direct control of the PC, and the user cannot control it directly, but it is in close relation with I_{qs}^* (equation 8) which is under the user control. Since the value of f_s is known in the controller, the user can adjust the desired value of f_s by setting the appropriate value of I_{qs}^* .

Table 1 The torque produced on the motor shaft with $I_{ds}^* = 3.6\text{A}$.

f_s [Hz]	T_{e0} [Nm]	T_e [Nm]		T_e/T_{e0}	
		$\varepsilon = 0.5$	$\varepsilon = 1.5$	$\varepsilon = 0.5$	$\varepsilon = 1.5$
0.25	0.82	0.39	1.19	0.4756	1.4512
0.50	1.63	0.78	2.38	0.4785	1.4601
0.75	2.45	1.25	3.61	0.5102	1.4735
1.00	3.26	1.68	4.76	0.5153	1.4601
1.25	4.08	2.04	5.88	0.5000	1.4412
1.50	4.91	2.45	6.61	0.4989	1.3490
1.75	5.71	2.80	7.65	0.4904	1.3398
2.00	6.53	3.20	8.49	0.4900	1.3002
2.25	7.34	3.53	9.39	0.4810	1.2793
2.50	8.16	3.76	10.2	0.4608	1.2500
2.75	8.98	4.13	11.1	0.4599	1.2361
3.00	9.79	4.48	12.0	0.4576	1.2257

All the results were collected immediately after the steady state has been reached. Between each two measurements, there was a pause, long enough to avoid the rotor heating. The measurements are considered short enough that the rotor resistance can be considered constant and equal to the previously determined R_r at room temperature.

In the second series, the three groups of measurements are repeated, but this time the reference for flux-producing current has been half of the nominal ($I_{ds}^* = I_{dsnom}/2 = 1,8\text{A}$) The results of this series are presented in Table 2 with the same meaning as in the previous one.

Table 2 The torque produced on the motor shaft with $I_{ds}^* = 1.8\text{A}$.

f_s [Hz]	T_{e0} [Nm]	T_e [Nm]		T_e/T_{e0}	
		$\varepsilon = 0.5$	$\varepsilon = 1.5$	$\varepsilon = 0.5$	$\varepsilon = 1.5$
0.25	0.20	0.11	0.29	0.5500	1.4500
0.50	0.41	0.22	0.57	0.5366	1.3902
0.75	0.61	0.35	0.82	0.5738	1.3443
1.00	0.82	0.52	1.02	0.6341	1.2439
1.25	1.02	0.69	1.14	0.6765	1.1176
1.50	1.22	0.85	1.32	0.6967	1.0820
1.75	1.43	1.02	1.44	0.7133	1.0070
2.00	1.63	1.24	1.55	0.7607	0.9509
2.25	1.84	1.40	1.69	0.7609	0.9185
2.50	2.04	1.55	1.86	0.7598	0.9118
2.75	2.24	1.70	2.01	0.7589	0.8973
3.00	2.45	1.83	2.18	0.7469	0.8898

The aim of the experiment has been to determine the ratio T_e/T_{e0} of the actual developed torque with detuned controller (T_e), and the torque that would be achieved if the controller were well tuned (T_{e0}). The ratio T_e/T_{e0} has been observed for the controller being out of tune in two ways:

- when the parameter R_r^* used in the controller has been half of the real value of the rotor resistance (R_r) in the machine ($\varepsilon = 0.5$), and
 - when R_r^* has been 50% higher than real R_r ($\varepsilon = 1.5$).
- Knowing the measured T_e/T_{e0} (shown on Figure 3) one can experimentally determine the saturation and detuning impact factor (χ_s and χ_d), and verify developed expressions for the detuned currents and flux (equations 13 to 19).

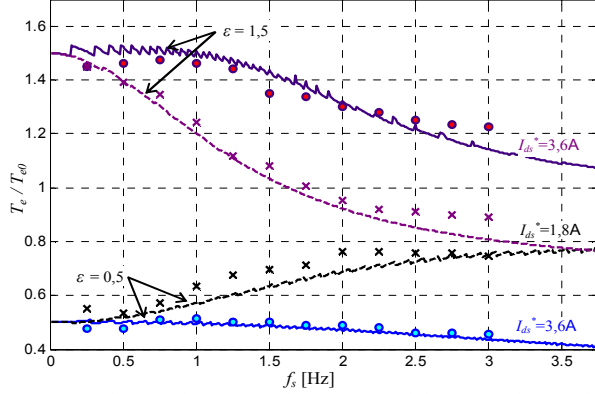


Figure 3 The ratio between real torque T_e and the torque that would exist in tuned drive T_{e0} . Curves are calculated, markers measured.

4.2 Influence of the R_r variation on the stator currents components

By measuring T_e/T_{e0} ratio, we actually measure $\chi_s^{3/2} \chi_d$ because we can get this product as $(T_e/(\varepsilon T_{e0}))^{1/2}$, according to equation (19). To verify the derived equations for I_{ds} current (13) we should split χ_s from χ_d . It can be done by a numeric procedure illustrated on Figure 4.

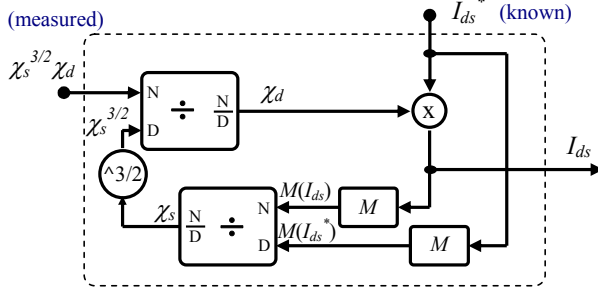


Figure 4 The numerical way to get measured relation between I_{ds} and I_{ds}^* from measured T_e/T_{e0}

The results are shown on Figure 5. The curves illustrate equation (13) and the markers are obtained by applying the numeric procedure mentioned above on the measured T_e/T_{e0} .

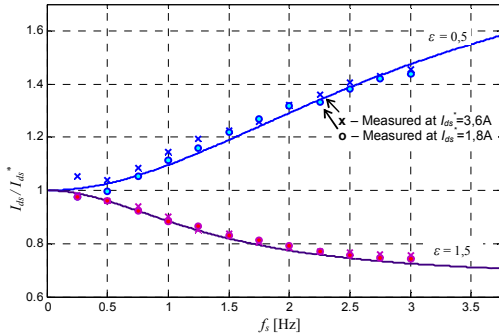


Figure 5 The real torque producing current I_{ds} in a detuned drive. The curves are calculated, markers are measured

4.3 Influence of the R_r variation on the rotor flux amplitude

Once we have the real value of I_{ds} , the rotor flux amplitude is easy to find as $\psi_r = M(I_{ds}) I_{ds}$. Figure 6 presents equation (17) (the curves), while the markers denote the values got from measured T_e/T_{e0} .

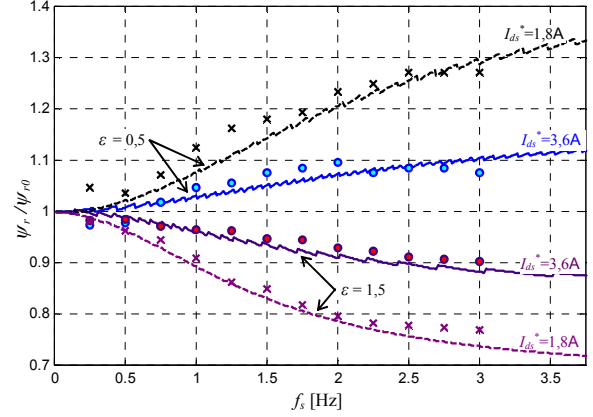


Figure 6 The real rotor flux ψ_r in a detuned drive. Curves are calculated, markers measured.

5 CONCLUSION

This paper shows the behavior of a detuned IFOC drive. When the value of the rotor resistance used in the model differs from the real value in the machine (due to the rotor heating or any other reason), the torque producing current and flux producing current are different from the commanded values even if the current controllers are ideal. The paper shows the analytical and graphical presentation of the real values of the currents, torque and flux.

The mutual inductance nonlinearity greatly affects system performance, and should be taken into account.

An experimental setup confirmed the analytical and numerical analysis.

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