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WIDE-BAND SPIRAL INDUCTOR MODEL PARAMETER EXTRACTION BASED ON GENETIC ALGORITHMS

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Abstract: *An automated procedure for parameter extraction of wide-band on-chip spiral inductor model is developed in the present paper. A computer program accepts the measured two-port S-parameters as input data and calculates the model parameter values using Genetic Algorithms. The proposed approach shows excellent agreement with the measured data over the whole frequency range for different geometries and technologies spiral inductors.*

Key Words: ***Parameter extraction, S-parameters, Planar spiral inductors, Genetic Algorithms (GA)***

1. INTRODUCTION

The on-chip spiral inductor is a very important constructive component of the contemporary CMOS microelectronics. In the CMOS SoC RFs the use of the spiral inductors in designs like VCOs, mixers, RF amplifiers, impedance-matching circuits is widespread. Many papers, devoted on the on-chip spiral inductors modeling and parameter extraction were published in the recent years. The wide-band spiral inductor model, presented in [1] has excellent accuracy compared to the measured results. The main advantage of this model is the frequency independence of its model parameters.

The model from [1] was widely accepted and several extraction procedures were published to aid the verification and the easy implementation in the microelectronic design. On the base of the model schematic from [1], many modifications of the model were developed to improve the accuracy of the model for different microelectronic technologies. Optimization procedures are often applied to aid the parameter extraction procedures [2], including the use of Genetic Algorithms (GA) [3]. There is a variety of examples for multiple variable optimizations in microelectronics based on the use of GA [4]. The geometric technology parameters for a given technology can be optimized to achieve given electrical characteristics [5] and the parameter extraction itself can be realized using GA for the spiral inductor model with frequency dependent series resistance [6, 7]. Accurate results are obtained using GA and Particle Swarm Optimization in the parameter extraction for the modified wide-band spiral inductor model compared to measured results for six-metal BiCMOS7 process octagonal inductors [8].

We propose an approach to wide-band spiral inductor model parameter extraction, based on GA. The MATLAB environment is chosen as an industry standard in the present paper for the parameter extraction procedure development, based on the GA toolbox [9]. The purpose function is designed to minimize the errors between the measured and the extracted *S*-parameters. Two sets of input *S*-parameters for two different technologies are used - 0.18-μm six-metal CMOS process input data set, published in [1] and 0.35-μm 2P4M CMOS process input data set, published in [10].

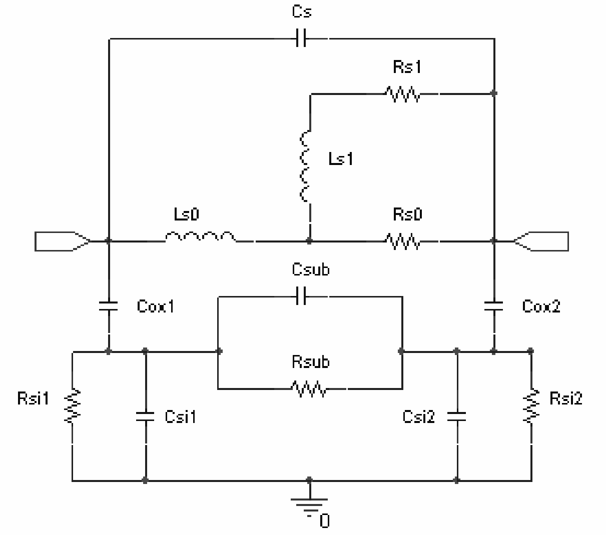

Fig. 1. *Wide-band spiral inductor model [1].*

2. PLANAR INDUCTOR CIRCUIT ANALYSIS

The wide-band spiral inductor model [1] is shown in Fig. 1. The model can be simplified from the analysis point of view, if presented using the impedance equivalent schematic shown in Fig. 2.

The following expressions can be written for the model shown in Fig. 2:

$$Z_{Ls0} = sL_{s0}; Z_{s1} = R_{s1} + sL_{s1};$$
$$Z_{Rs0} = R_{s0}; Z_{Cs} = 1/sC_s \qquad (1)$$

$$Z_s = \left(Z_{Ls0} + Z_{s1} \parallel Z_{Rs0}\right) \parallel Z_{Cs} \quad (2)$$

$$Z_{ox1,2} = 1/sC_{ox1,2} \quad (3)$$

$$Z_{Rsi1,2} = R_{si1,2}\,;\; Z_{Csi1,2} = 1/sC_{si1,2} \quad (4)$$

$$Z_{si1,2} = Z_{Rsi1,2} \parallel Z_{Csi1,2} \quad (5)$$

$$Z_{Rsub} = R_{sub}\,;\; Z_{Csub} = 1/sC_{sub} \quad (6)$$

$$Z_{sub} = Z_{Rsub} \parallel Z_{Csub} \quad (7)$$

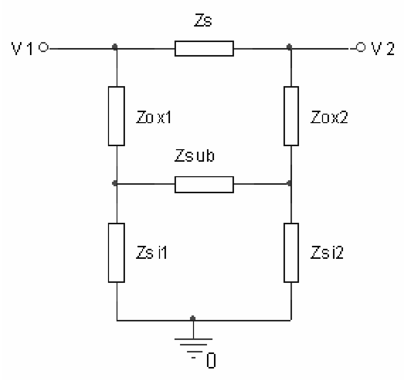

Fig. 2. *Impedance equivalent schematic of the spiral inductor model from Fig. 1.*

The Y-parameters can be found using equations (2), (3), (5) and (7). According to the two-port definition for the Y-parameters, Y_{11} and Y_{21} are defined for the case when $V_2 = 0$, and Y_{22} and Y_{12} – for the case when $V_1 = 0$.

If Z_{eqY11} is the equivalent impedance of the circuit for the case when $V_2 = 0$ (Fig. 2) the following expression for Y_{11} can be written [3]:

$$Y_{11} = V_1/Z_{eqY11} \quad (8)$$

Because of the symmetry of the model in Fig. 2, for the equivalent impedance Z_{eqY22} and Y_{22} in the case when $V_1 = 0$, the following expression is valid:

$$Y_{22} = V_2/Z_{eqY22} \quad (9)$$

Using the currents I_s and I_{Zox2}, for the parameter Y_{21} in the case when $V_2 = 0$, the following expression is valid [3]:

$$Y_{21} = -\left(I_s + I_{Zox2}\right)/V_1 \quad (10)$$

Because of the symmetry of the model in Fig. 2, for Y_{12} in the case when $V_1 = 0$, the following expression is valid, using equation (11):

$$Y_{12} = -\left(I_s + I_{Zox1}\right)/V_2 \quad (11)$$

3. PARAMETER EXTRACTION PROCEDURE USING GA

The first step is to determine the ranges of variation for the model parameters (Fig. 1). The ranges can vary for the different microelectronic technologies. The following expressions can be used [2]:

$$R_{s0} \approx \Re\left(-1/Y_{21}\right) \quad (12)$$

$$L_{s0} \approx \Im\left(-1/Y_{21}\right)/\omega \quad (13)$$

$$R_{Zsi} \approx \Re\left(1/\left(Y_{11} + Y_{21}\right)\right) \quad (14)$$

$$C_{Zsi} \approx \Im\left(1/\left(Y_{11} + Y_{21}\right)\right)/\omega \quad (15)$$

$$C_{ox} = \max\left(C_{Zsi}\right) \quad (16)$$

$$R_{si} = \max\left(R_{Zsi}\right) \quad (17)$$

$$C_{si} \approx \frac{C_{ox}C_{Zsi}}{C_{ox} + C_{Zsi}} \quad (18)$$

The expressions (12) and (13) are used for low frequencies and the expressions (14), (15), (16) and (17) are used for frequencies, where the capacitors C_s and C_{sub} and the resistor R_{sub} (Fig. 1) can be neglected – low and middle frequencies. The ranges of variation for the model parameters R_{s1} and L_{s1} are chosen to be the same as for the model parameters R_{s0} and L_{s0} correspondingly. The variation ranges for the model parameters R_{sub} and C_{sub} depend on the used technology and the conductance of the substrate. The variation ranges for the 0.18-µm six-metal CMOS process input data set [1] are given in Table 1 and for the 0.35-µm 2P4M CMOS process input data set [10] – in Table 2 (second and third columns).

Table 1. *Variation ranges and extracted values for the model parameters of spiral inductor with geometry 2.5x60x14.5x2 implemented using 0.18-µm six-metal CMOS process [1]*

Model Parameter	Min Value	Max Value	Ext Value
R_{s0}, Ω	1	50	4.58
R_{s1}, Ω	1	50	10.2
L_{s0}, nH	1	20	1.83
L_{s1}, nH	1	20	3.8
C_{ox1}, fF	50	500	72.24
C_{ox2}, fF	50	500	61.09
R_{si1}, Ω	50	1000	146.73
R_{si2}, Ω	50	1000	145.2
C_{si1}, fF	10	100	38.34
C_{si2}, fF	10	100	37.14
R_{sub}, Ω	500	50000	39431
C_{sub}, fF	50	1000	50.67
C_s, fF	10	500	10.52

Once the basic circuit analysis and Y-parameter expressions are built, a GA approach can be applied to extract the model parameter values. The idea is to compare the measured S-parameters and the S-parameters, obtained when the model parameter values are varied in a certain range. The actual comparison is done in the purpose function, which compares the absolute value for every frequency point of every two-port S-parameter (real and imaginary part) in one expression, using the sum of the least squares values:

$$G_{fun} = \sum_{i=1}^{n}\left\{\Re\left[S_{jk}\left(f_i\right)\right] - \Re\left[S^{(m)}{}_{jk}\left(f_i\right)\right]\right\}^2 + \sum_{i=1}^{n}\left\{\Im\left[S_{jk}\left(f_i\right)\right] - \Im\left[S^{(m)}{}_{jk}\left(f_i\right)\right]\right\}^2 , \quad (19)$$

where $j,k = 1,2$;

$S_{jk}^{(m)}$ - measured S-parameters;

S_{jk} - simulated S-parameters, using the extracted model parameters;

n – number of the measured/simulated frequency points.

Table 2. *Variation ranges and extracted values for the model parameters of spiral inductor Ind2 implemented using 0.35-µm 2P4M CMOS process (geometry not mentioned) [10]*

Model Parameter	Min Value	Max Value	Ext Value
R_{s0}, Ω	1	50	2.87
R_{s1}, Ω	1	50	2.37
L_{s0}, nH	1	20	1.38
L_{s1}, nH	1	20	1
C_{ox1}, fF	50	500	500
C_{ox2}, fF	50	500	500
R_{si1}, Ω	50	5000	5000
R_{si2}, Ω	50	5000	5000
C_{si1}, fF	10	100	10
C_{si2}, fF	10	100	10
R_{sub}, Ω	500	50000	500
C_{sub}, fF	50	1000	105.78
C_s, fF	10	500	125.92

The initial (measured) *S*-parameters are obtained from .xls file, and the current *S*-parameters are obtained, converting the current *Y*-parameters, obtained from expressions (8), (9), (10) and (11). The variation ranges for the model parameters are fixed in the *FieldD* matrix [9], where the number of columns corresponds to the number of variables, which are optimized by the GA.

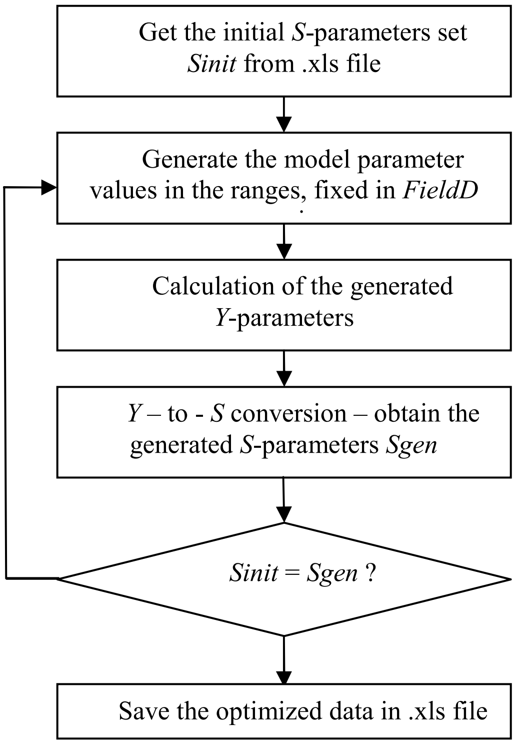

Fig. 3. *The sequence of steps, used in the GA-based spiral inductor parameter extraction procedure.*

The values of the model parameters are ranged in the predefined ranges (Table 1, Table 2). The whole optimization process is controlled by the value of the purpose function (19), whose purpose is to minimize the difference between the initial and the generated *S*-parameters (Fig. 3).

The GA optimization is done using GA Toolbox [9] in MATLAB for 100 iterations, generation gap of 0.7 for population of 200 individuals. The function body contains two *for* cycles. The first cycle runs the calculations for every frequency point until the end (*DATA_ROWS*), and the second cycle runs the calculations for every individual for a given frequency point until the end (*Nind*). The MATLAB code of the body of the parameter extraction procedure is shown below:

```
   for i = 1:DATA_ROWS %The first cycle runs the
calculations for every frequency point
            s = j*2*pi*frequency(i);

         for ix = 1:Nind %The second cycle run the
calculations for every individual

            ZLs0 = s.*Ls0(ix);
            Zs1 = Rs1(ix) + s.*Ls1(ix);
            ZRs0 = Rs0(ix);
            ZCs = 1./(s.*Cs(ix));

            ZsRL = ZLs0 + ((Zs1.*ZRs0)./(Zs1 + ZRs0));
            Zs = ZsRL*ZCs/(ZsRL + ZCs);

            Zox1 = 1./(s.*Cox1(ix));
            Zox2 = 1./(s.*Cox2(ix));

            ZRsi1 = Rsi1(ix);
            ZCsi1 = 1./(s.*Csi1(ix));
            Zsi1 = (ZRsi1.*ZCsi1)./(ZRsi1 + ZCsi1);

            ZRsi2 = Rsi2(ix);
            ZCsi2 = 1./(s.*Csi2(ix));
            Zsi2 = (ZRsi2.*ZCsi2)./(ZRsi2 + ZCsi2);

            ZRsub = Rsub(ix);
            ZCsub = 1./(s.*Csub(ix));
            Zsub = (ZRsub.*ZCsub)./(ZRsub + ZCsub);

            %Calculate Y11, V1 = 1, V2 = 0
            Zox2si2 = (Zox2.*Zsi2)./(Zox2 + Zsi2);
            Zsubox2si2 = Zox2si2 + Zsub;
            Zsubox2si2si1 = (Zsi1.*Zsubox2si2)./(Zsi1 +
Zsubox2si2);
            Zsubox2si2si1ox1 = Zox1 + Zsubox2si2si1;
            ZeqY11 = (Zs.*Zsubox2si2si1ox1)./(Zs +
Zsubox2si2si1ox1);
            Y11(ix, 1) = U1./ZeqY11;

            %Calculate Y22, V1 = 0, V2 = 1
            Zox1si1 = (Zox1.*Zsi1)./(Zox1 + Zsi1);
            Zsubox1si1 = Zox1si1 + Zsub;
            Zsubox1si1si2 = (Zsi2.*Zsubox1si1)./(Zsi2 +
Zsubox1si1);
            Zsubox1si1si2ox2 = Zox2 + Zsubox1si1si2;
            ZeqY22 = (Zs.*Zsubox1si1si2ox2)./(Zs +
Zsubox1si1si2ox2);
            Y22(ix, 1) = U2./ZeqY22;

            Ysub = 1./Zsub;
            Yox1 = 1./Zox1;
            Yox2 = 1./Zox2;
```

```
        Ysi1 = 1./Zsi1;
        Ysi2 = 1./Zsi2;

        %Calculate Y12, V1 = 0, V2 = 1
        %Expressions taken from symmetry
considerations with Y21
        UZox1 = (Yox2.*Ysub)./((Yox2 +
Ysi2).*(Ysub + Yox1 + Ysi1) + Ysub.*(Yox1 + Ysi1));
        IZox1 = UZox1./Zox1;
        Is = U2./Zs;
        I1 = Is + IZox1;
        Y12(ix, 1) = -I1./U2;

        %Calculate Y21, V1 = 1, V2 = 0
        %Expressions taken from the symbolic
extraction of Y21
        UZox2 = (Yox1.*Ysub)./((Yox1 +
Ysi1).*(Ysub + Yox2 + Ysi2) + Ysub.*(Yox2 + Ysi2));
        IZox2 = UZox2./Zox2;
        Is = U1./Zs;
        I2 = Is + IZox2;
        Y21(ix, 1) = -I2./U1;

         %Y->S parameters conversion
        y_params(1, 1, ix) = Y11(ix, 1);
        y_params(1, 2, ix) = Y12(ix, 1);
        y_params(2, 1, ix) = Y21(ix, 1);
        y_params(2, 2, ix) = Y22(ix, 1);

        s_params(:,:,ix) = y2s(y_params(:,:,ix),50);

        S11(ix, 1) = s_params(1, 1, ix);
        S12(ix, 1) = s_params(1, 2, ix);
        S21(ix, 1) = s_params(2, 1, ix);
        S22(ix, 1) = s_params(2, 2, ix);

    end

  %Least squares sum of the difference between the
real and imaginary parts of the required and the current
S-parameters
        g_fun = g_fun + (abs(real(S11)) -
abs(real(S11_req(i)))).^2 + (abs(imag(S11)) -
abs(imag(S11_req(i)))).^2 + (abs(real(S22)) -
abs(real(S22_req(i)))).^2 + (abs(imag(S22)) -
abs(imag(S22_req(i)))).^2 + (abs(real(S12)) -
abs(real(S12_req(i)))).^2 + (abs(imag(S12)) -
abs(imag(S12_req(i)))).^2 + (abs(real(S21)) -
abs(real(S21_req(i)))).^2 + (abs(imag(S21)) -
abs(imag(S21_req(i)))).^2;

    end
```

4. RESULTS

The presented wide-band on-chip spiral inductor model parameter extraction procedure is verified according to the published data in [1, 10]. The root mean square (RMS) error over the frequency range for the real and the imaginary parts of the extracted and the measured *S*-parameters is used to estimate the accuracy of the extraction procedure for various geometry and technology RF spiral inductors. The full range RMS error is calculated using the formula (20):

$$RMSErr_S = 100 \times \sqrt{\sum_{i=1}^{n} \frac{\left(S_{jk}^{(m)} - S_{jk}\right)^2}{n}}\,, \qquad (20)$$

where $j,k = 1,2$;

$S_{jk}^{(m)}$ - measured *S*-parameters;

S_{jk} - simulated *S*-parameters, using the extracted model parameters;

n – number of the measured/simulated frequency points.

The obtained results for the full range RMS errors after applying the extraction procedure are given in Table 3 for the real and the imaginary parts of the extracted *S*-parameters in respect to the measured *S*-parameters, published in [1], [10]. The spiral inductors *I*1 and *I*2 are with geometry 2.5x60x14.5x2 (2.5 - number of turns, 60 - inner radius (μm), 14.5 - metal width (μm), 2 - spacing (μm)) and 3.5x60x14.5x2 respectively, implemented, using 0.18-μm six-metal CMOS process (Table 3) [1]. The spiral inductors *I*3 (*Ind*1 [10]) and *I*4 (*Ind*2 [10]) are implemented, using 0.35-μm 2P4M CMOS process and their geometries are not specified (Table 3) [10]. The RMS error estimation in Table 3 is valid for *I*1 and *I*2 with measured *S*-parameters in the range (100MHz – 20GHz); measured data for *I*3 in the range (50MHz – 10GHz); measured data for *I*4 in the range (50MHz – 16GHz).

Table 3. *Full range RMS errors after the parameter extraction in respect to the measured S-parameters, published in [1] and [10]*

Spiral Inductor	Full range RMS errors, *S*-parameters real parts, %			
	S_{11r}	S_{12r}	S_{21r}	S_{22r}
*I*1 [1]	1	1.76	1.54	0.96
*I*2 [1]	2.99	1.85	1.8	2
*I*3 [10]	1.56	2.17	2.17	2.86
*I*4 [10]	1.9	1.1	1.12	1.79
Spiral Inductor	Full range RMS errors, *S*-parameters imaginary parts %			
	S_{11i}	S_{12i}	S_{21i}	S_{22i}
*I*1[1]	2.08	1.07	0.82	1.5
*I*2[1]	3.42	1.2	1.13	1.83
*I*3[10]	5.47	1.16	1.23	2.32
*I*4[10]	5.83	2.12	2.23	5.99

A comparison between the measured and extracted results for the *Q*-factor and the *S*-parameters S_{11} and S_{12} is given in Fig. 4, Fig. 5 and Fig. 6. The obtained model parameter values for *I*1 and *I*4 are given in Table 1 and Table 2 (fourth column) correspondingly.

5. CONCLUSION

Different geometries spiral inductors, implemented using 0.18-μm six-metal CMOS process [1] and 0.35-μm 2P4M CMOS process [10] are investigated in the present paper and their model parameters are extracted, using GA based parameter extraction procedure. The obtained results for the extracted *S*-parameters have excellent accuracy in respect to the measured data, published in [1] and [10]. The full range RMS errors for the real and the imaginary parts of the S-parameters are up to 6%.

The extraction time is around 5 minutes for 100 iterations on Intel Core 2 Duo 1.8 GHz processor.

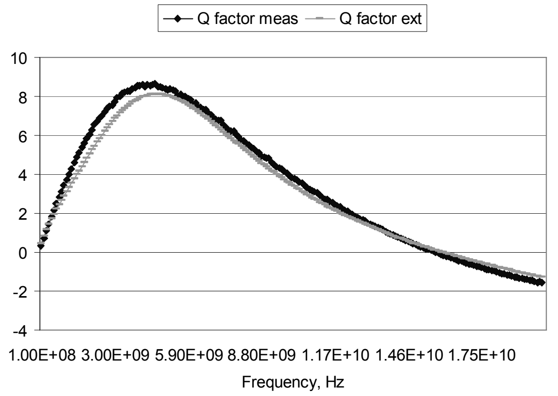

Fig. 4. *Comparison between the measured Q-factor (Q factor meas) and the extracted Q-factor (Q-factor ext) for spiral inductor I1 [1].*

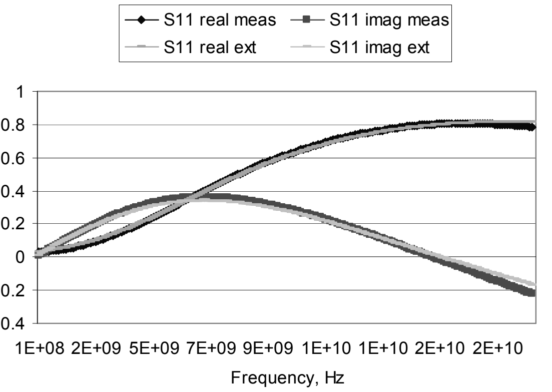

Fig. 5. *Comparison between the measured real and imaginary parts of* S_{11} *(*S_{11} *real meas,* S_{11} *imag meas) and the extracted real and imaginary parts of* S_{11} *(*S_{11} *real ext,* S_{11} *imag ext) for spiral inductor* I1 *[1].*

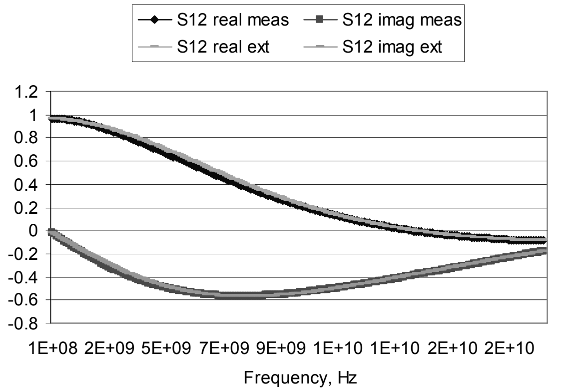

Fig. 6. *Comparison between the measured real and imaginary parts of* S_{12} *(*S_{12} *real meas,* S_{12} *imag meas) and the extracted real and imaginary parts of* S_{12} *(*S_{12} *real ext,* S_{12} *imag ext) for spiral inductor* I1 *[1].*

6. ACKNWLEDGEMENT

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