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HARMONIC ANALYSIS OF THE BOOST CONVERTER WITH BIFURCATION BEHAVIOUR

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Abstract: *A boost converter controlled by naturally-sampled constant frequency pulse-width modulation in a discontinuous conduction mode of operation is analysed in the frequency-domain. Several steady-state responses are recognised by varying the dc input voltage only. A reasonable accordance between results of computer simulation and experiments is obtained.*

Key-words: ***Boost converter, Frequency-domain analysis, Harmonics.***

1. INTRODUCTION

One of the most active research topics in power electronics is the analysis of all steady-state responses of the boost converter [1-3]. It is motivated by the widespread application of this type of dc converters as a basic building block of single-phase power factor correction circuits.

The correct design of the boost converter assumes that all possible modes of operation and their dependences on variation of converter parameters are known in advance. In this way it is possible to avoid certain undesirable performance during service such as occurrence of increased dc output ripple or audible sound.

A series of papers has been published recently investigating possible steady-state responses with special attention paid to the chaotic behaviour of the boost converter [4-7].

In our study of boost converter a special attention is paid to the calculation of harmonic content of state variables. Also, we are interested how to present characteristic sub-harmonic in such a way that ensure easy identification of the steady-state responses of the boost converter.

2. STATE EQUATIONS

The equivalent circuit of the realised boost converter is shown in Fig.1. The proposed mathematical model was verified in [8]. The discontinuous conduction mode is assumed, i.e. there are three changes of power circuit topology in a period of operation $T=1/f$,

a) $V1$ closed, $V2$ open
b) $V1$ open, $V2$ closed
c) $V1$ and $V2$ open

Therefore, the boost converter can be modelled by piece-wise linear time-varying network which passes in a period of operation T through three different configuration.

a) When $u_{\mathrm{ramp}} \geq u_i$, the controlled switch V_1 is in the ON-state and the diode V_2 is in the OFF-state. The state equations are:

$$\frac{du_C}{dt} = \frac{1}{CR_d}u_C \qquad \frac{di_L}{dt} = \frac{1}{L}\left(E - Ri_L\right) \tag{1}$$

b) When $u_{\mathrm{ramp}} < u_i$, $i_L>0$, the controlled switch V_1 is in the OFF-state and the diode V_2 is in the ON-state. The state equations are:

$$\frac{du_C}{dt} = \frac{1}{C}\left(i_L - \frac{u_C}{R_d}\right) \qquad \frac{di_L}{dt} = \frac{1}{L}\left(E - Ri_L - u_C\right) \tag{2}$$

c) When $u_{\mathrm{ramp}} < u_i$, $i_L=0$, the controlled switch V_1 and the diode V_2 are in the OFF-state. The state equations are:

$$\frac{du_C}{dt} = \frac{1}{CR_d}u_C \qquad i_L = 0 \tag{3}$$

According to Fig.1 the input voltage of the operational amplifier OA2 $u_i(t)$ is given by

$$u_i = \frac{-R_4}{R_5}U_{ref} + \left(1 + \frac{R_4}{R_5}\right)\cdot\frac{R_2}{R_1 + R_2}u_C \tag{4}$$

The external saw-tooth ramp voltage:

$$u_{ramp} = \frac{3}{T}t + 0{,}7 \ \text{[V]} \tag{5}$$

In all simulations the fourth-order Runge-Kutta method with the same step size h=50 ns was used for the numerical integration of state equations. The parameters of converter which were taken as constant are: R_1=22kΩ, R_2=1.22kΩ, R_4=2.7MΩ, R_5=3.9kΩ, R=1.05Ω, L=698µH, C=470µF, R_d=56Ω, U_{ref}=5V and T=500µs with the dc input voltage E varying only.

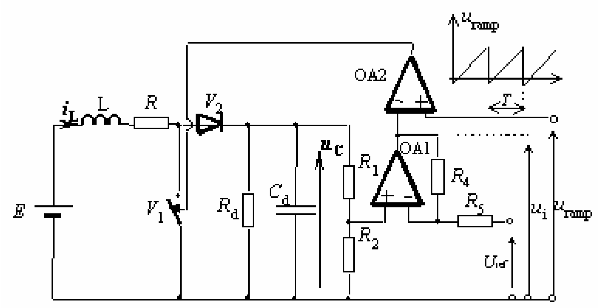


Fig 1. *The equivalent circuit of the boost converter.*

3. FREQUENCY-DOMAIN ANALYSIS

The time-domain analysis enables us to solve several important design problems, like the determination of voltage and current stresses of converter devices. But, there are other important design problems not traceable by the time-domain analysis.

Let us assume, for instance, that the boost converter is intended to be used as a dc power supply. Then, in additional to the given static and dynamic deviation of the dc output voltage the most important data determining quality of the dc power supply are the maximum allowable ac component of the current injected into the dc supply as well as the maximum allowable ac component of the voltage applied to the dc load.

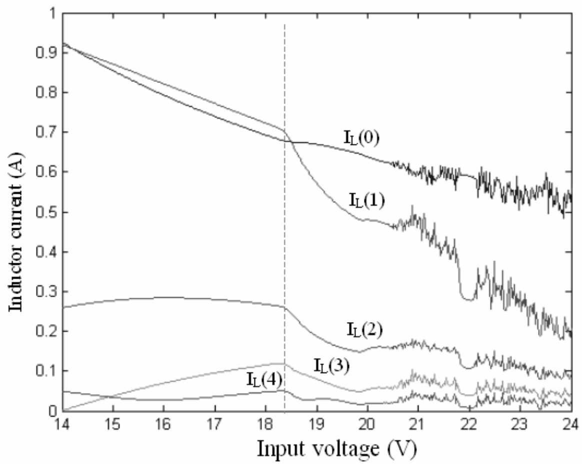


Fig. 2. *DC component (n=0) and the first four harmonics of inductor current (n=1,2,3,4).*

It means that these ac components ought to be known to the designer in order to design or not the additional input filter and/or the output filter.

For this purpose it is not sufficient to know the ac component of the input current (inductor current i_L) as well as the ac component of the output voltage (capacitor voltage u_C) only, but their harmonic content. Here, under the term harmonic content we mean a set of RMS values of the variable under question at each significant frequency. Therefore, it is necessary to carry out the harmonic analysis of state variables for all possible steady states.

In our case, harmonic analysis was carried out by using a standard software package of MATLAB 6.0. The computed waveforms of inductor current i_L and capacitor voltage u_C, from extended time interval [$50T$-$170T$], were used as input data. The results obtained for the first four harmonics of the inductor current $I_L(nf)$, n=1,2,3,4 as well as the dc component $I_L(0)$ versus the dc input voltage E are shown in Fig. 2.

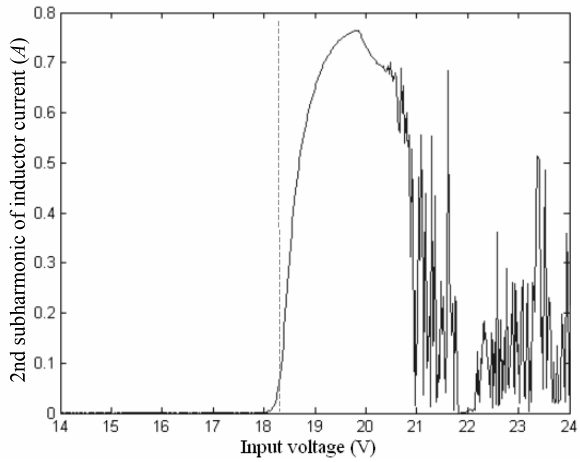


Fig. 3. *Second sub-harmonic of inductor current (n=1/2).*

After the first bifurcation, i.e. transition from period-one operation to period-two operation at E=18.2V, the second sub-harmonic (harmonic of order n=1/2) increases rapidly as shown in Fig. 3. That explains a significant decrease of all harmonics after the first bifurcation, as noticeable in Fig. 2.

The second bifurcation, i.e. transition from period-two operation to period-four operation starts at E=19.8V, and the fourth sub-harmonic (harmonic of order n=1/4) increases, as shown in Fig. 4.

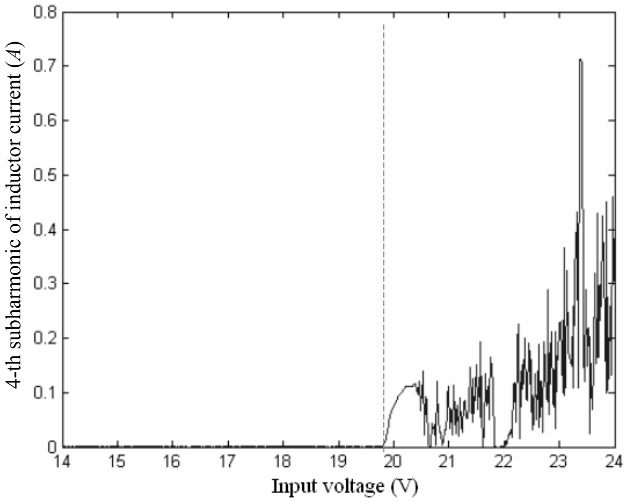


Fig. 4. *Fourth sub-harmonic of inductor current (n=1/4).*

During the chaotic mode of operation the RMS values of harmonics as well as sub-harmonics, as expected, change irregularly. In the vicinity of the dc input voltage E=22V, period-three operation is noticeable in Figures 2.-4. The third sub-harmonic (harmonic of order n=1/3) versus the dc input voltage E is displayed in Fig. 5.

In order to verify the results of harmonic analysis we have carried out measurements of the first four significant harmonics and of the second sub-harmonic of state variables at particular values of the dc input voltage.

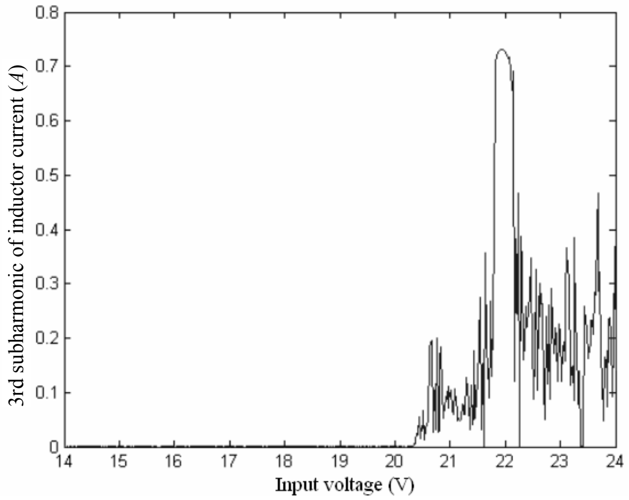


Fig 5. *Third sub-harmonic of inductor current (n=1/3).*

The measurements were obtained by using a Tektronix oscilloscope series TDS224 with additional module for fast Fourier transformation TDS2MM. The measurement data were calculated with the software package of WSTRO. The computed results and the results obtained by measurements at E=19V (period-two operation) are shown in Table 1. The qualitative accordance between results is observed. Also, a process of period-doublings is recognised as one of the known route to chaos.

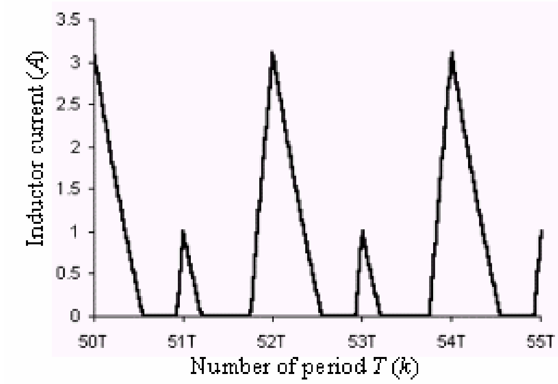
Table 1. *Harmonic content of state variables at E=19V*

Harmonic order n	Inductor current I_L (n) RMS; %	
	Measured values	Computed values
1/2	**0.72 A; 100%**	**0.65 A; 100%**
1	**0.54 A; 74.6%**	**0.57 A; 88.1%**
2	**0.17 A; 24.2%**	**0.19 A; 28.9%**
3	**0.05 A; 7.5 %**	**0.08 A; 12.5 %**
4	**0.04 A; 6.2 %**	**0.03 A; 4.6 %**

Harmonic order n	Capacitor voltage U_C (n) RMS; %	
	Measured values	Computed values
1/2	**182.6 mV; 100%**	**182.4 mV; 100%**
1	**86.9 mV; 47.6%**	**78.5 mV; 50.6%**
2	**22.7 mV; 12.4%**	**17.9 mV; 11.5%**
3	**16.2 mV; 8.8 %**	**9.6 mV; 6.1 %**
4	**10.6 mV; 5.8 %**	**4.9 mV; 3.1 %**

Bifurcation points, i.e. the values of dc input voltage E at which the new period of operation occurs, are obtained easily from the corresponding bifurcation diagram of sub-harmonics (Fig.2-5.). To verify the proposed mathematical model of the boost converter it is necessary to compare the results of simulation with the results of measurements. At the moment, we are not able to obtain the bifurcation diagram of characteristic sub-harmonic by measurements. So, the direct comparison was made by inspection and qualitative comparison of the measured and computed harmonic content of inductor current for characteristic steady-state.

a)



b)

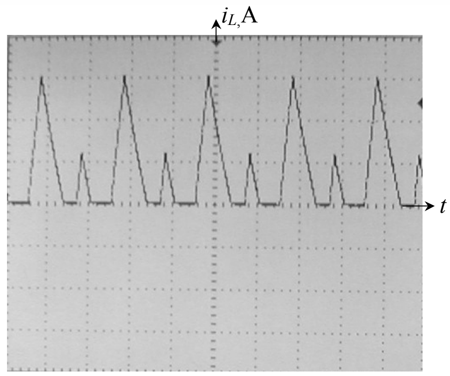


Figure 6. *Period two-operation (E=19V)*
a) Computed inductor current
b) Measured inductor current
(1A/div,500μs/div)

As an example, the characteristic steady-state is period-two operation and waveforms of inductor current obtained by measurement and simulation are shown in Fig.6. In this way, for all bifurcation points given in Table 2, like 17.9V; 19.7V; 20V; etc. are determined. These results correspond very well to the results obtained from the bifurcation diagram of characteristic sub-harmonics.

Table 2. *Regions of steady-state responses*

Steady state responses	***E*,V**	
	Measured values	Computed values
Period-one operation	14-17.9	14-18.2
Period-two operation	17.9-19.7	18.2-19.8
Period-four operation	19.7-20	19.8-20.2
Period-eight operation	-	20.2-20.3
Chaos	20-21.8 21.9-24	20.3-21.8 21.9-24
Period-three operation	21.8-21.9	21.8-21.9

4. CONCLUSIONS

The boost converter has been studied from bifurcation point of view. The bifurcation diagrams of the characteristic harmonic for period doubling route to chaos provide a clear picture about boundaries between bifurcation and in that way ensure the designers to choose converter parameters for the desired operating regime. A good agreement between simulation results and measurement implies the validity of the chosen mathematical model.

5. REFERENCES

[1] J.H.B.Deane, "Chaos in current-mode controlled boost DC-DC converter", *IEEE Transactions on Circuits and Systems-1*, vol.39, pp. 680-683, 1996.

[2] C.K.Tse, "Flip bifurcation and chaos in three-state boost switching regulators"*, IEEE Transactions on Circuits and Systems-1*, vol.41, pp. 16-23, no.1, 1994.

[3] S.Banerjee, K.Chakrabarty, "Nonlinear modeling and bifurcation in the boost converter", *IEEE Trans. on Power Electronic,* vol.13, no.2, March 1998, pp.252-260.

[4] J.H.B.Deane, D.C.Hamill, "Instability, subharmonics and chaos in power electronic systems", *IEEE Transactions on Power Electronics*, vol.5, no.3, 1990, pp. 260-268.

[5] W.GY.Chan, C.K.Tse, "Study of bifurcation and chaos in current-programmed DC-DC boost converters: From quasi-periodicity to period-doubling", *IEEE Transactions on Circuits and Systems*, vol.44, pp. 1129-1142, no.12, 1997.

[6] A.El Aroudi, L.Benadero,E.Toribo, G.Olivar: "Hopf bifurcation and chaos from torus breakdown in a PWM voltage-controlled DC-DC boost converter", *IEEE Transactions on Circuits and Systems*, vol.46, no.11, 2000, pp. 1374-1382.

[7] I.Zafrani, S.Ben-Yankov, "A chaos model of subharmonic oscillations in current mode PWM boost converter", *IEEE Power Electronic Special Conference, PESC 1995,* pp. 1-7.

[8] D.Pelin, K.Miličević: "Parameter space maps of the boost converter", 14*th* *International Symposium on Power Electronics-Ee 2007*, Novi Sad, Republic of Serbia, November 2007, no. T1-1.1, pp. 1.-5.

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