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# THE USE OF PRONY ANALYSIS AND PADÉ APPROXIMATION FOR THE IDENTIFICATION OF TRANSFER FUNCTIONS OF EXCITATION SYSTEM COMPONENTS

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**Abstract**: *The paper presents the new, enhanced way of the use of Prony analysis, combined with Padé approximation, for the transfer functions identification. Transfer functions of interests are related to automatic voltage regulator (AVR) and excitation system. In the presented analysis the influence of field voltage recorded signal corruption with noise is successfully overcame. By the use of standard first order synchronous generator model, it is possible to extract exact generator model parameters, using Padé approximation. After that step, the transfer function of automatic voltage regulator (AVR) is derived using similar technique. Finally, derived generator and AVR transfer functions are verified by Matlab/Simulink simulation.*

**Key Words:** ***Prony analysis/identification/synchronous generator***

## 1. INTRODUCTION

To fulfill the needs imposed by small signal stability studies, it is necessary to have the appropriate equivalent models of generators embedded in the power system. Once the power system low frequency oscillations occur, the operators deploy appropriate mitigation measures to damp them.

When a linear time invariant system is described by a differential equation, it is common approach to derive impulse response using Laplace transform. If the rational transfer function expression can be obtained than it can be factored in a number of poles. In such case the inverse transform is represented as a sum of exponential functions of time, and this approach is foundation of the Prony analysis [1].

The Prony analysis has been shown for a long time to be a viable technique to model a linear sum of damped complex exponentials to signals that are uniformly sampled. It is not only a powerful signal analysis technique but also a system identification method, which is widely used in the areas of power system electromechanical oscillation [2, 3, 4, 5]. As compared to other oscillatory signal analysis techniques such as Fourier analysis, Prony analysis has the advantage of being able to estimate damping coefficients in addition to frequency, phase angle and amplitude. It is also possible to take into account offset and drift, up to some degree. Furthermore it is the most suitable to identify a reduced-order model of a high-order system both, in time and frequency domain.

In this paper the Prony analysis is applied to the parameter identification problem, related to small signal transfer functions of different devices involved in voltage control loop of synchronous generators. The case study generator is 210 MW, 247 MVA, 15,75 kV round rotor turbogenerator commissioned in thermal power plant.

## 2. THE IDENTIFICATION CONCEPT

The goal of the presented research is to determine the AVR and generator small signal transfer function without intrusive testing of the unit. The classical transfer function determining procedure is giving elusive result, strongly dependent on internal transducers' delays and heavily polluted by measurement noise. To remove noise it is necessary to filter the input (field voltage) and the output data (terminal voltage or reactive power). The recorded and appropriately filtered field voltage and the terminal voltage or reactive power data, are used as inputs to Prony analysis. The derived, least square error fitted, time responses are then converted into *s* – domain functions, using the appropriate symbolic calculation software. The corresponding small signal transfer function is then determined as the ratio of Laplace transforms of output and input signal. In such way it is possible to get the voltage transfer function of synchronous generator (SG) with AVR plus excitation, as well as reactive response characteristics of total closed loop system and generator.

The main problem associated with briefly described non-intrusive testing approach is the existence of different time delays associated with different measuring sensors. As a consequence, the estimated transfer function contains parasitic zeros and poles. To overcome this problem, i.e., to remove parasitic poles and zeros of transfer function, the widely accepted practice is to limit the frequency range of expected poles and zeros to the fastest acceptable dynamics.

In this research, however, another approach is taken based on the use of Padé approximant of transfer function in low frequency domaine. Padé approximant is approximation of a function by a rational function of given order, and as such is well suited to the transfer function approximation problem in cases where the number of poles and zeros is generally known and the parasitic poles and zeroes are located generally in high frequency range. The analysis presented further in the paper demonstrates full applicability of the proposed method for intended purpose.

## 3. CASE STUDY

To accomplish synchronous generator parameter identification task, it is necessary to improve the recorded signals processing in such manner that the noise corrupted field voltage signal can be used without heavy filtering. Heavy filtering of the field voltage introduces frequency dependent phase shift and due that there is a need to filter all identification signals with same filter. With this approach, however, some other medium frequency information can be lost.

The proposed analysis is performed in two stages.

The first stage is recording the appropriate step response of the generating unit reactive power and field voltage following the change in AVR reference voltage. Reactive power is chosen to be the original output signal due to easy conversion to SG terminal voltage change and due to relatively small content of measurement noise. The input step size is 0.5% of the rated reference voltage. Recorded reactive power trace is converted into terminal voltage change trace by multiplication with the corresponding reactance, in order to suppress the noise. Sampling frequency during recording was 2 kHz. The generator was delivering at 67 MVAr, 173 MW at 15.75 kV prior to applying step change in AVR reference voltage. . The resulting step increase in reactive power was 9MVAr, i.e., from 67 MVAr to 76 MVAr.

At the second stage the modified Prony approach is applied. Based on the fact that all time responses within closed loop have the same pole pattern [6, 7], the generator terminal voltage response was fitted using exponential function (1),

$$V_t(t) = a \cdot e^{-b \cdot t} \cdot \cos(d \cdot t + f) + e1 \cdot e^{-g \cdot t} + h, \qquad (1)$$

where coefficients *a*,*b*,*c*,*d*,*e*1,*g* and *h* are derived using least square error (LSE) fitting procedure. The recorded terminal voltage response (black) and fitted terminal voltage response (red) are shown in Fig. 1. The base voltage value is 15.75 kV and the base power is 247 MVA. Excellent agreement between the two can be observed in Fig 1.

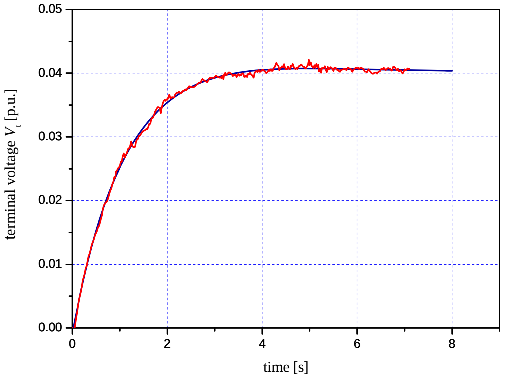

Fig. 1. *Recorded (black trace) and exponentially approximated (red trace) generator terminal voltage change step response*

In this way, the important common parameters for all loop signals are derived, i.e. the coefficients *b*, *d* and *g* values are common for all mentioned loop signals. Having estimated these coefficients, the exponential approximation of the field voltage change can be applied more efficiently. The field voltage signal is, typically, much more corrupted with noise than other recorded loop signals. The field voltage step response is approximated by sum of exponential functions (2) taking into account previously estimated values for coefficients *b*, *d* and *g*.

$$V_f(t) = a_x \cdot e^{-0.6819264 \cdot t} \cdot \cos(25.04543 \cdot t + f_x) + \\ + e1_x \cdot e^{-1.437443 \cdot t} + h_x + l \cdot e^{-k \cdot t} \,. \qquad (2)$$

Additional exponential term compared to (1) was introduced in (2) in order to get better fitting of field voltage signal. By doing this the original pole pattern of the transfer function was slightly changed. The LSE fitting procedure was therefore applied again and the coefficients $a_x$, $f_x$, $e1_x$, $h_x$, $l$ and $k$ determined. The recorded field voltage response (blue) and estimated field voltage response (red) are shown in Fig. 2. The base field voltage is 350 V.

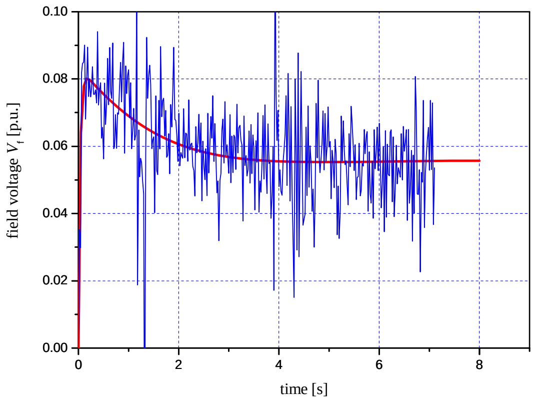

Fig. 2. *Recorded (blue trace) and exponentially approximated (red trace) generator field voltage step response*

From Fig. 2 it can be seen that the heavily corrupted (by noise) measured field voltage response is cleaned up using the modified Prony method. Additionally, there was no need for additional low pass filtering of the field voltage signal and consequently there was no phase shift introduced. It is important to mention that the substantial part of recorded signals has to be selected, in order to get rid of extra nonlinearities and signal corruption.

Having all coefficients of (1) and (2) determined, now it is possible to get the corresponding Laplace transforms. Laplace transforms of (1) and (2) are divided in order to get the generator transfer function (field voltage to terminal voltage change), corresponding to previously described operating point. The resulting transfer function was rather complex and Padé approximation was used to simplify it and convert into corresponding first order transfer function (the simplest low frequency small signal SG model is in the form of the single pole transfer function). [8, 9] The time constant and gain of the resulting first order SG transfer function are dependent on SG operating point. Additional pair of complex poles, considering the third order small signal SG model, is located in higher frequency range. Having in mind slow changes of voltage that are of interest in this case study, they can be neglected. The target Padé approximant is of the first order and it is developed around zero value of *s*.

The Padé approximant of the order [m/n] is defined as the rational function which has equal derivatives with the original function up to the highest possible order at the chosen point [10]. All modern numerical program tools have implemented algorithm for this rational approximation.

The Padé approximant of the first order of the generator transfer function is given by (3),

$$G_g = \frac{7.8862}{130.88 \cdot s + 94.024}. \tag{3}$$

Laplace transform of (1) divided by Heaviside step function with the weight coefficient equal to 0.005 p.u. is expressed as closed loop transfer function reference to terminal voltage (4),

$$G = \frac{-0.059632 \cdot s^3 + 2.2341 \cdot s^2 + 4.1025 \cdot s + 2.2092}{2.5 \cdot s^3 + 7.0031 \cdot s^2 + 6.5412 \cdot s + 2.3576}. \tag{4}$$

Following some algebraic manipulations of the closed loop transfer function, according to (5)

$$G_{AVR} = \frac{G}{G_g(1-G)}, \tag{5}$$

the AVR + excitation system transfer function (TF) is derived. The resulting TF was then replaced by Padé approximant of the second order, around zero value of *s*. The final form of the derived TF is given by (6),

$$G_{AVR} = \frac{57.967(2.0304 \cdot s^2 + 5.0962 \cdot s + 3.0928)}{8.1671 \cdot s^2 + 14.98 \cdot s + 1.0097}. \tag{6}$$

## 4. RESULT VALIDATION

To prove the validity of abovementioned approximations, and especially the validity of (6), it is necessary to perform simulation check of derived approximated transfer functions of AVR + excitation system and generator.

The corresponding simulation is performed in Simulink/Matlab environment, using the block diagram shown in Fig. 3. The results of simulation are shown in Fig. 4.

The red line corresponds to derived terminal voltage step response using Prony method, while the blue line corresponds to simulated closed loop terminal voltage step response, performed with Padé approximants. The very good agreement between the two responses that can be seen in Fig. 4 validates the proposed use of Prony analysis and Padé approximation for estimation of AVR + excitation system + generator transfer function.

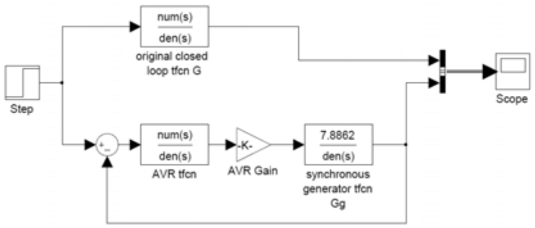

Fig. 3. *The Simulink block diagram of the verification circuit*

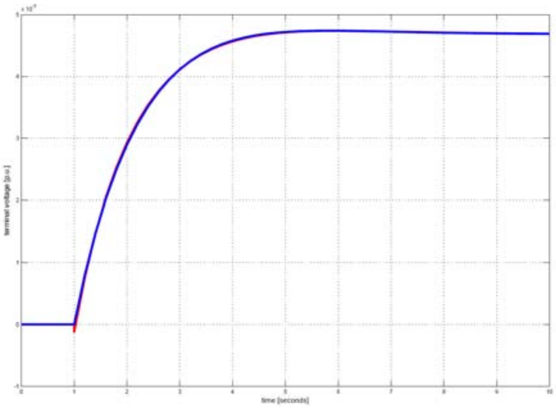

Fig. 4. *Terminal voltage change traces in p.u. as outputs of the circuits shown in Fig. 3. Red trace is Prony aproximation of the terminal voltage change and blue trace is simulated closed loop output*

## 5. CONCLUSIONS

The paper presented very efficient tool for estimation of transfer function of SG + AVR + excitation system based on Prony analysis and Padé approximation .

The method successfully overcomes the influence of recorded field voltage corruption with noise without the need for additional filtering and consequently partial loss of signal information.

The proposed identification concept is demonstrated on a case study of a real 210 MW turbogenerator and associated AVR and excitation system. The results are validated using simulations in Matlab Simulink environment.

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