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PID AUTOTUNING – RELAY FEEDBACK METHOD (ÅSTRÖM & HÄGGLUND METHOD)

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Abstract: *In this project is performed an analysis and check of Relay feedback method for adjustment of parameters of PI regulator. The object of control is the temperature in the box. A 1,2 kW power heater is used as an actuator. Component used for measuring temperature is 47kΩ NTC resistor. Prior to experimental check, a theoretical analysis is done and mathematical model of the method is written. The algorithm for the project is written in C programmable language for developing board STM32F4-Discovery. Checkup of the system stability is also performed during the experiment.*

Key Words: ***PI regulator/Autotuning/STM32F4***

1. INTRODUCTION

Most of the industrial processes demands certain type of regulation for achieving best performances. There are several ways for realization of process regulation, but the most famous and widespread is PID (*proportional-integral-differential)* regulation. The aim of regulation is not just about achieving the best performances, but keeping the system stable. The type of regulation can be divided in to two sorts:

- *Open-loop control*
- *Closed-loop control*

It is necessary to know the nature of the process for both sorts of regulation. The algorithm for the first sort is usually defined during the design of the process, and it is not affected by the noise. Therefore, all the variables that effect the process are defined. In case of changed working conditions, in comparison to designed, by the process, a serious deviation can occur between desired and actual behavior of the system. Second sort of regulation allows changes of managing algorithm depending on measured interferences during the process. This way it is possible to successfully regulate the process even when the conditions of itself changes.

Quality of the regulator depends on the choice of algorithm and set parameters. Choice of the algorithm for the regulator is based on the nature of the regulated process, while the parameters of the regulator are set based upon the transfer function. If the transfer function of the process is known, parameters of the regulator can be precisely calculated. For the majority of the process in the industry, the transfer function can not be exactly defined, therefore for the regulator parameters setup of such processes is developed several methods. These methods are based on theoretical analysis and practical experiments. Best known are: Ziegler-Nichols, Chien-Hrones-Reswick, Astrom-Hagglund (Relay feedback), Tyreus-Luyben, Setpoint overshoot, Good gain, etc.

This paper represents detail analysis and practically tested the method of Astrom-Hagglund (Relay feedback). With the application of STM32F407VG is realized an algorithm of the mentioned method for determination of regulator parameters and regulator itself. Regulated variable is temperature.

2. ABOUT THIS METHOD

In 1995. Astrom and Hagglund have developed a method for regulator auto-tuning with relay feedback. Several of the modern methods are based on Astrom and Hagglund method, such as: Luyben (2001), Huang (2005), Sung and Lee (2006), Ma and Zhu (2006) etc. Main goal is to bring the system to continuous oscillation and to, based on the amplitude and period of oscillations, calculate maximal amplification of the system, as well as the delay. This method is vastly applicable in the industry for several reasons:

- Experiment is performed with feedback, wich results in maintenance of the regulated variable of the system within set value
- It is not necessary to know the time constant of the system before the experiment is preformed, for the determination of time sample.

General block scheme is presented in the figure below:

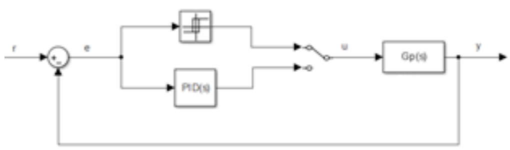

Fig. 1. *Relay in feedback*

With the application of this method it is possible to determine the parameters of highly nonlinear processes, for the real variable is maintained within the set value, which can be interpreted with high enough accuracy as a linear area (area of the value). At the same time, this is the basis of this method. Experiment can be performed with microcontroller, but it is also possible to do it manually. It can be performed by following next 5 steps (book Auto tuning of PID controllers – Cheng Ching YU, page 25):

1. *Bring the system to steady state*
2. *Make a small (e.g. 5%) increase in the manipulated input. The magnitude of change depends on the process sensitivities and allowable deviations in the controlled output. Typical values are between 3 and 10%.*
3. *As soon as the output crosses the set point, the manipulated input is switched to the opposite position (e.g. –5% change from the original value).*
4. *Repeat step 2 until sustained oscillation is observed.*
5. *Read off ultimate period from the cycling and the value of the amplitude*

Meaning, if the system is at a steady state, and the value of the managing variable is 100, it is required to set the value of the managing variable to 103, and keep that value until the fault becomes negative. When that occurs, the value of the managing variable should be reduced to 97, and so forth until constant oscillations are achieved.

To have a better understanding of the confusing first step (what is *steady state*?), next example will be observed:

Case No. 1.

The temperature of the water is to be held at 70°C. Power of the heater is 1kW, nominal voltage is 230V. For example, let's say that steady state of the water at 40°C is achieved with 100W power of the heater. The first step is accomplished, ie. steady state is accomplished. In the next step, we increase the power of the heater by 3%, namely to 103W. This power remains until the fault is positive (measured temperature of the water is less than required 70°C). As soon as the fault becomes negative, the power of the heater should be set at 97W and so forth to continue with the oscillations. A questions rises: What if the real value of the temperature does not exceed the set value with the 3% increase, and a new steady state is established at 45°C? What is the next step then? This is the essence of this method, that the experiment is performed within the environment of the working point, ie. set value. This way, the system will not be set at steady state at 40°C, but at 78°C for example, and therefore by manipulating the power of the defined value, the system will truly bring about to oscillate.

That being said, the first step should be defined more precisely:

1. *Bring the system to a steady state close enough to the set value, so that by manipulating* the managing variable within *3%-10% of the value, the system oscillates.*

During the performance of the experiment, after the established constant oscillations, it is required to measure the amplitude and the period of the oscillation.

In some industrial processes it is not possible to provide constant regulation of the managing variable, for example in the before mentioned case No. 1 it means that we cannot set the desired value of the heater, but only to turn it on or off. This is also acceptable, but the estimation of the maximal amplification and frequency will be slightly worse.

This paper will be focused exactly on the second case, and at the next figure is presented the algorithm of the method.

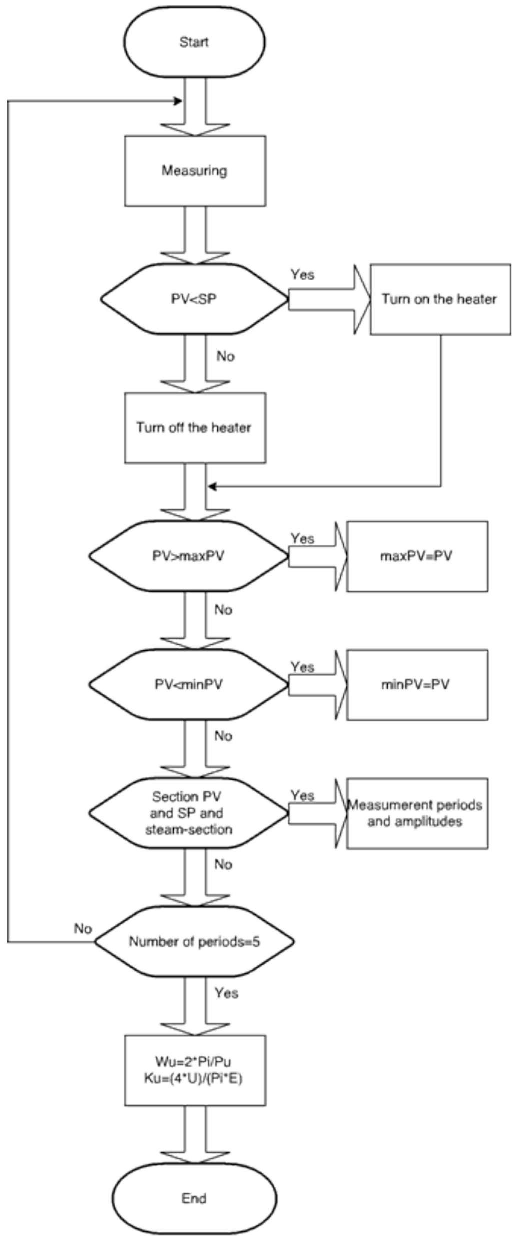

Fig. 2. *Algorithm*

3. FORMULAS NECESSARY FOR THE RELAY FEEDBACK METHOD OF DETERMINING THE PARAMETERS

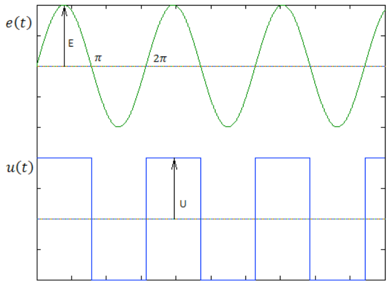

Fig. 3. *Input and output signals*

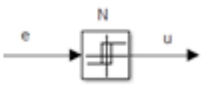

Fig. 4. *Relay simulink block*

Transfer function of the nonlinear system presented on the figure 4. in general is:

$$N(t) = \frac{u(t)}{e(t)} \qquad (1)$$

Where the variables are:

$e(t)$ – input variable (fault signal – difference between set and measured value)

$u(t)$ – output variable

$N(t)$ – Transfer function of the relay

Let's assume that the input signal is sinusoidal shape with the amplitude E and angular frequency. With this approximation it is possible to describe fault signal with enough accuracy.

$$e(t) = E\sin(\omega t) \qquad (2)$$

Based on a Figure 3 function $u(t)$ can have following values:

$$u(t) = \begin{cases} U, 0 \le \omega t \le \pi \\ -U, \pi \le \omega t \le 2\pi \end{cases} \qquad (3)$$

Output function of the relay can be described with Fourier series:

$$u(t) = \frac{1}{2}a_0 + \sum_{k=1}^{\infty}(a_k \cos k\omega t + b_k \sin k\omega t) \qquad (4)$$

Coefficients a_0 and a_k have a value 0 cause:

$$a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} u(t)\cos(k\omega t)\, d(\omega t) = \frac{2}{\pi}U\int_0^{\pi}\cos(k\omega t)\, d(\omega t) = 0 \qquad (5)$$

Coefficient b_k is:

$$b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} u(t)\sin(k\omega t)\, d(\omega t) = \frac{2}{\pi}U\int_0^{\pi}\sin(k\omega t)\, d(\omega t) \qquad (6)$$

Neglecting higher harmonics $k \neq 1$, b_k is:

$$b_k = \frac{4U}{\pi} \qquad (7)$$

therefore the output function is:

$$u(t) = \frac{4U}{\pi}\sin(\omega t) \qquad (8)$$

Based on these equations, transfer function of the relay becomes:

$$N = \frac{4U}{\pi E} \qquad (9)$$

After the determination of the transfer function of the relay, an analysis of the system from the Fig. 5. can be performed.

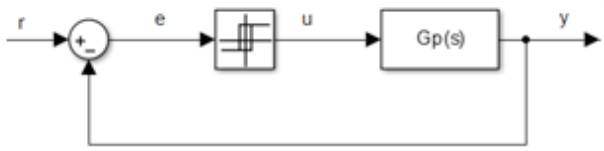

Fig. 5. *Relay feedback control (block schematic)*

The system from a Fig. 5. can be described with following equations:

$$E = -Y \qquad (10)$$

$$U = N \cdot E \qquad (11)$$

$$Y = G_p \cdot U \qquad (12)$$

$$G_p = -\frac{1}{N} \qquad (13)$$

Conclusion is that the amplification of the function G_p is exactly $-\frac{1}{N}$. The state of the system is not known for this amplification (ie. The system is stable, unstable or marginally stable). The aim is to determine maximal amplification with the system staying stable. This amplification fits the amplification of the marginally stable system. New variable K_{HS} is introduced into the last equation from above, changing it into:

$$G_p = -\frac{K_{HS}}{N} \qquad (14)$$

Applying the Niquist's criteria of stability, we get the maximal amplification:

$$K_{HS} = N \qquad (15)$$

which makes maximal frequency the oscillation frequency

$$\omega_k = \omega_{osc} \qquad (16)$$

To sum up, maximal amplification and frequency of the system are:

$$K_{HS} = \frac{4U}{\pi E} \qquad (17)$$

$$\omega_k = \frac{2\pi}{P_u} \qquad (18)$$

where P_u presents period of oscillations.

Based upon determined amplification and frequency, with the measurement of time of the first up growth (D), it is possible to construct transfer function of the system with the following equations:

$$\tau = \frac{\tan(\pi - D\omega_u)}{\omega_u} \tag{19}$$

$$K_p = \frac{\sqrt{(\tau\omega_u)^2 + 1}}{K_u} \tag{20}$$

First series system with transport delay:

$$G_p(s) = \frac{K_p e^{-Ds}}{\tau s + 1} \tag{21}$$

4. SIMULATION (MATLAB-SIMULINK)

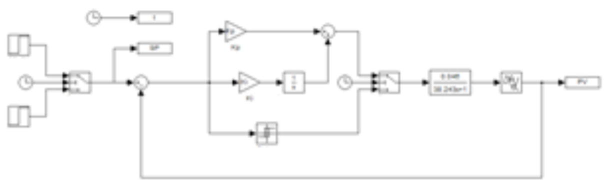

Fig. 6. Block schematic

To be able to analyze the experiment, it is necessary to perform simulation so that comparison of the work of algorithm can be done. For simulation of work, the transfer function is the one that is estimated with practical analysis.

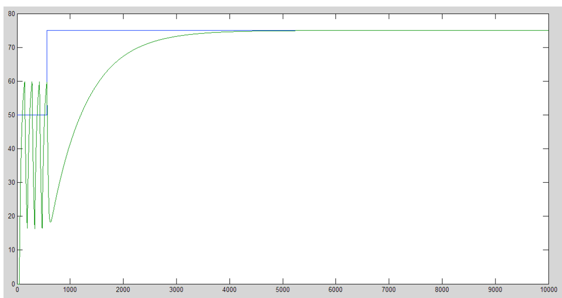

Fig. 7. *Response of the system*

At the figure above, it is possible to see the area of oscillations and working area of PI regulator. Similar graphics should be anticipated during practical work.

5. EXPERIMENT

Experiment will be used to check theoretical assumptions and claims. Algorithm and necessary calculations will be performed with microcontroller STM32F407 (STM32F4-Discovery development board). Visualization of the experiment and the results are realized with STMStudio v3.4.

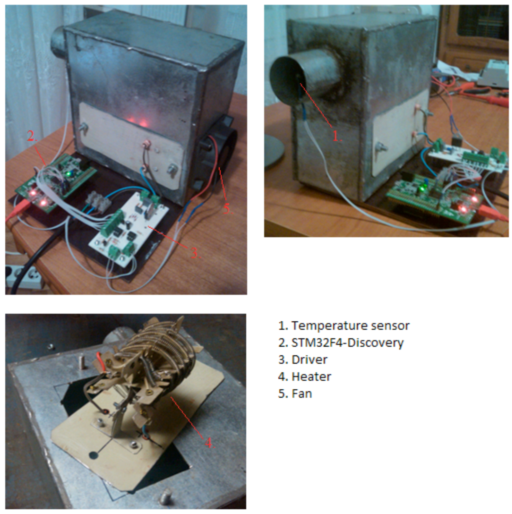

Fig. 8. *Model appearance*

The aim of the experiment is to regulate the temperature of the air in the exhaust branch. Sensor used to measure the temperature is 47kΩ NTC resistor, on temperature of 25°C. As a heat source is used a 1,2kW heater from a hairdryer. PC fan is used to increase inertness of the system, but also as a second managing variable during later tests.

To be able to perform the experiment it is necessary to provide certain electronics which suits demands defined in mathematical analysis. Basic demands are power regulation of the heater and temperature measurement. As it is said before, the power of the heater as a managing variable should be square-shaped during the oscillations of the system. Bearing this in mind, regulation of power of the heater can be solved with the application of relay, but for further testing, the driver is composed in such a way to fulfill all the demands as explained in theoretical section. Regulation of power can be managed with phase control.

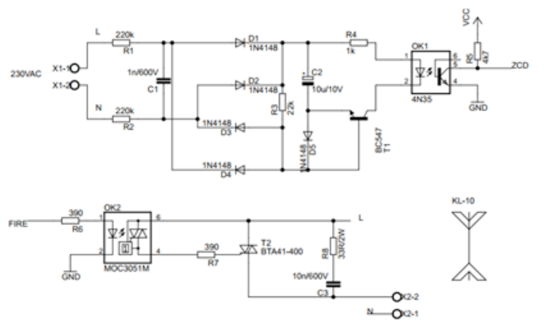

Fig. 9. *Schematic of power regulation of the heater*

The figure above presents two different sections: "Zero cross detector" and "Phase regulation". First block detects crossing of grid voltage through "zero", which is needed for measurement of the angle for switching on triac. The line ZCD directly leads to the controller. Second block contains of triac, driver MOC3051 (replacement FOD4218) and snubber circuit (RC). The chosen driver for the triac has no integrated zero crossing detector circuit necessary for phase regulation. The line FIRE is connected to the microcontroller.

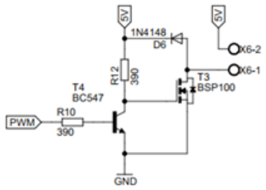

Fig. 10. *Schematic for fan regulation*

This section of the driver enables regulation of speed of the fan, but during this experiment it will not be used – speed of fan will be constant.

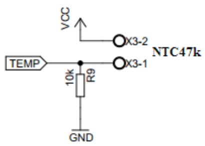

Fig. 11. *Schematic for temperature measurement*

The line TEMP is led to an analog pin of the microcontroller, and filter and linearization is realized within software.

After all the physical preparations comes the adjustment of the microcontroller for the realization of algorithm.

After switching on the model, controller first sets in motion the algorithm for determining parameters for PI regulator for given set point (SP) at 50°C. When this phase is done, SP is set to 75°C and PI regulator is turned on with calculated coefficients. Graphic and the results are being monitored on PC in STMStudio program. The determination of parameters for the regulator is a 5 oscillations cycle, of which the average one of amplitude and period is considered, excluding the first oscillation. After these calculations, the values of the parameters for PI regulator are defined based on the following table:

Table 1. *Kp, Ki, Kd COEFFICIENTS*

	K_p	K_i	K_d
P	$0,5K_u$	-	-
PI	$0,4K_u$	$0,5\frac{K_u}{P_u}$	-
PID	$0,6K_u$	$0,8\frac{K_u}{P_u}$	$0,075K_uP_u$

The point of turning on or off the heater depends on the crossing of the measured value of the temperature (PV) through SP$\pm$1°C. Hysteresis of 1°C is set for elimination of the influence of the noise. It is repeated every 1s.

6. RESULTS AND ANALYSIS

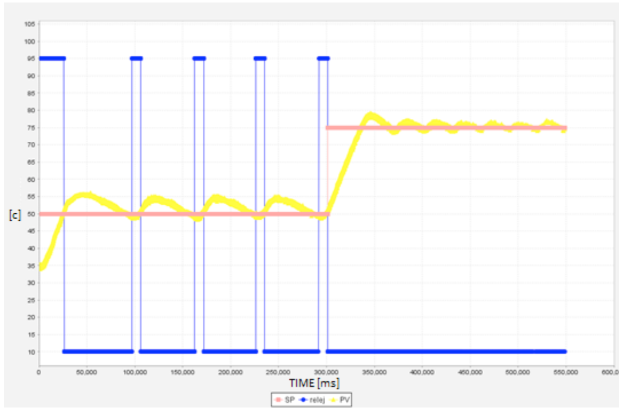

Fig. 12. *Chart of method and PI regulation*

In graphics above it can be seen that duration of Relay Feedback method is around 300s, ie. 5min. With parameters determined, PI regulator is turned on and reference is set at 75°C. In figure 5.6 are respectively shown amplitudes and periods of all 5 oscillations, as well as calculated parameters for PI regulator. The fault value of PI regulator can be read off figure 5.7, which is +1°C i -0,75°C. The percentage of this error is +1,33% and -1%.

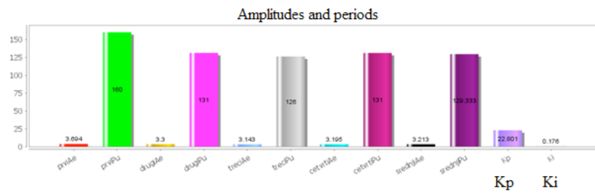

Fig. 13. *Amplitudes and periods*

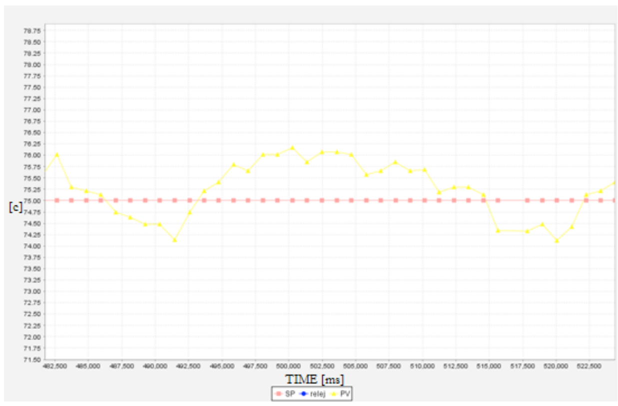

Fig. 14. *PI regulator – Error*

During the experiment a checkup of stability of the system is done. Parameter estimation is executed in nominal conditions (fan powered with 5VDC). After the parameters were set, PI regulator was turned on also in nominal conditions. After 350s, the fan was turned off and the response was monitored. The conclusion was derived that the system has returned to its original state, but with increased fault. Then after 700s the fan is turned on to 12VDC, after which is noted that the system has returned to its original state faster. Within these parameters of fan working it is observed that the system is stable, bearing in mind the definition of system stability: System is stable, if it returns to its original state after external disturbances.

Reminder, a system would be considered unstable if the amplitudes of these oscillations would increase.

Fig. 15. *Stability test*

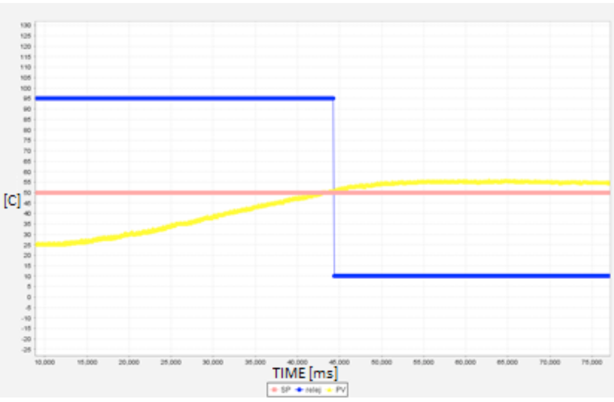

Fig. 16. *Temperature rise*

From the Fig. 16. it can be read the value D as 42,5s (time necessary for the temperature to rise to do max). Based on this time and calculated values for, it is possible to derive transfer function of the system as:

$$G(s) = \frac{0.046e^{-42.5s}}{38243.s + 1} \qquad (22)$$

Based on determined transfer function a Nyquist diagram is:

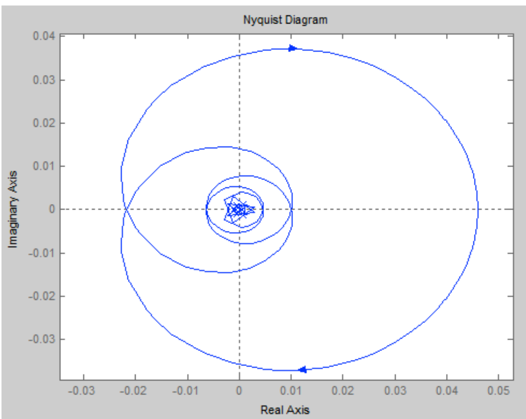

Fig. 17. *Nyquist diagram*

7. CONCLUSION AND FUTURE WORK

In this paper is analyzed and experimentally checked Relay Feedback method for setting PID regulator parameters. Method is not tested in its original formulation, more precisely the system did not oscillate by changing the power of the heater by ±3%-10%, but by turning the heater on and off, which also gave satisfactory results. Conclusion can be made that with finer oscillation can be achieved even better results. This type of parameter determination has a wide application in the industry.

Figure 12, gives us, besides designated variables, approximate power of stationary faults of the system at 50°C. They provide median power of the heater:

$$P_{qstac} = \frac{T_{on}}{T} \cdot P = \frac{10}{60} \cdot 1200 = 200W \qquad (23)$$

These faults are possible to measure at various temperatures and draw system fault characteristics depending on the temperature.

Using relay with hysteresis and power control of the heater for finer oscillations presents further way of testing this method. Also, PID regulator will be used instead of PI used in this experiment. To maximize the speed of the response, the power of the heater will be determined during the oscillations and that will be used for drainage of system near stationary spot, where PID will be turned on. This principle is known as "feed forward". Also, in the future is planned to be introduced fan regulation, for simulation of industrial systems with two managing variables. Beside mentioned advancements, the method will be subjected to tests in higher speed systems, with smaller delays.

8. REFERENCES

[1] K. J. Astrom and T. Hagglund (1984), "*Automatic Tuning of Simple Regulators with Specifications on Phase and Amplitude Margins*", Pergamon Press Ltd., Great Britain

[2] Cheng-Ching YU (2006), "*Autotuning of PID Controllers, A Relay Feedback Approach 2nd Edition*", Springer Science, Germany

[3] Jietae Lee and Su Whan Sung, "*Integrals of Relay Feedback Responses for Extracting Process Information*", Departman of Chemical Engineering Kyungpook National University Daegu, Korea. url=http://chem.engr.utc.edu/engr329/Lab-manual/relay-feedback.htm

[4] Dingyu Xue, Yang Quan Chen and Derek P.Atherton (2007), "*Linear Feedback Control*" ,Charter 6

[5] Roman Prokop, Jiri Korbel and Zdenka Prokopova, "*Relay-Based Autotuning with Algebraic Control Design*" ,Czech Republic

[6] D. Marčetić (2012), "*Mikroprocesorsko upravljanje energetskim pretvaračima*" , FTN izdavaštvo, Novi Sad, Srbija

[7] N. Perić, Z. Vukić, M. Baotić, N. Mišković (2014) "*Automatsko upravljanje 2014/2015*", Zavod za automatiku i računalno inženjerstvo, Hrvatska