Paper No. T6.3, pp. 1-4

18th INTERNATIONAL SYMPOSIUM on
POWER ELECTRONICS - Ee 2015

NOVI SAD, SERBIA, October 28th - 30th, 2015

SWITCHING SIMULATION APPLYING EXPLICIT RUNGE – KUTTA METHODS

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Abstract: *Software programs which are applied for simulating the dynamic behaviour of modern electric power systems are based on different numerical integration methods or on time differential equations' solutions. This paper presents the application of explicit Runge – Kutta methods for the simulation of a test circuit with a diode and its comparison with several other methods. The accuracy and stability have been tested for the switching processes in this study.*

Key Words: ***Accuracy, Comparison, Explicit Runge – Kutta methods, Stability, Switching mode***

1. INTRODUCTION

Electric power systems have been changed in the recent years. The main changes are connected to the place of electric power generation, the electrical energy transmission and the energy consumption. The major parts in the electric networks took the power electronic systems. The new trend of the electric power generation with renewable sources and HVDC transmissions requires DC network parts. Nevertheless, due to the fact that most electrical networks' parts are operating with AC currents, the challenges are to apply the right power electronic devices [1].

In order to investigate the behaviour of entire electric network involving power electronics, lot of simulations have to be done to study the transients occurring during any switching process. There are lot of procedures based on different numerical integration methods or on time differential equations' solutions. Each of those methods has its own advantages and disadvantages. Which numerical integration method or which time differential equation will be applied, depends on the specific exploring case [1].

In this paper, explicit Runge – Kutta methods are applied to simulate a diode switching mode in a simple RC circuit. Four different explicit Runge – Kutta methods [2] – [6] have been applied; 1st order-, 2nd order-, 3rd order- and 4th order Runge – Kutta method. Results of the simulations are compared with the different applied numerical integration methods or time differential equations, comparing the accuracy and stability.

2. EXPLICIT RUNGE – KUTTA METHODS

Comparing to the other numerical integration methods, explicit Runge – Kutta methods are especially convenient for the first order differential equations solving,

$$\mathrm{d}x/\mathrm{d}t = f\left(t,\ x\right). \tag{1}$$

These methods are one step numerical integration methods, which means that they apply one previous value in order to obtain a better one. Compared to the other observed methods, explicit Runge – Kutta method need less integration steps, during the iterations [2].

The form of the methods is,

$$\begin{gathered} x_{n+1} = x_n + h\sum_{i=1}^{\mathrm{s}} b_i k_i, \\ t_{n+1} = t_n + h, \end{gathered} \tag{2}$$

with

$$k_i = f\left(t_n + c_i h,\ x_n + h\sum_{j=1}^{\mathrm{s}} a_{ij} k_j\right), \tag{3}$$

where *a*, *b* and *c* denote the integration coefficients, *h*, *k*, s, *t* and *x* denote the time step size, integration stage, number of integration stages, time and the unknown function, respectively [3] – [6].

3. APPLYING EXPLICIT RUNGE – KUTTA METHODS IN A TEST CIRCUIT

The analyzed explicit Runge – Kutta methods and the other methods, applied for comparisons, have been applied on simulating a test circuit, which contains a diode as shown in Fig. 1.

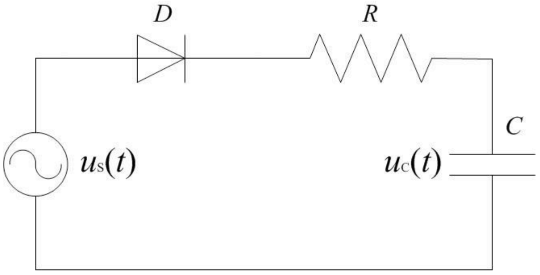

Fig. 1. *Test circuit.*

The circuit parameters are:

$$\begin{aligned} &u_{\mathrm{S}}(t) = \mathrm{U_{sm}}\sin(\omega t), && R = 5\ \Omega, \\ &\mathrm{U_{sm}} = \sqrt{2}\cdot 230\ \mathrm{V}, && C = 5\ \mathrm{mF}, \\ &u_{\mathrm{C}}(0) = 0, && f = 50\ \mathrm{Hz}. \end{aligned} \tag{4}$$

The circuit has been simulated applying the analyzed methods for a time step size of $\Delta t = 0.00005\ \mathrm{s}$ in order to calculate the capacitor voltage. In the same time, the same value of capacitor voltage was calculated applying trapezoidal integration formula in the "Simulink" program and the modified trapezoidal simulation in the "LTspice" program. All obtained results are shown graphically in Fig. 2.

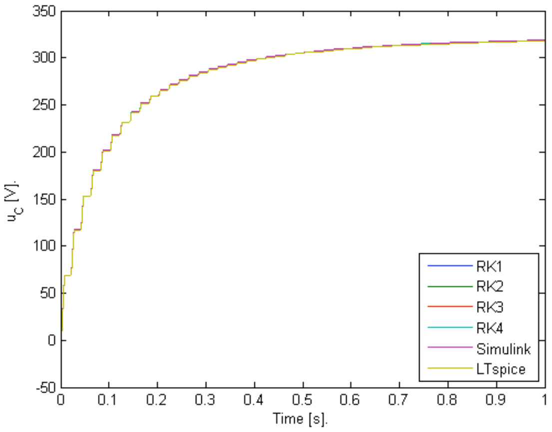

Fig. 2. *Simulation results applying all four orders of RK method, "Simulink" and "LTspice".*

As expected and as it can be seen in Fig. 2, there is no significant differences between compared results. The differences may be noticed only by zooming presented curves or by calculation of the deviations.

In order to verify all obtained results, three more methods for solving the same problem were adopted.

First, a partial analytical solution of the circuit presented in Fig. 1, replacing the diode with a very small resistance, $\mathrm{R_{dfor}} = 0.001\ \Omega$ in forward bias and a very big resistance, $\mathrm{R_{dinv}} = 10^9\ \Omega$ of inverse bias.

According to the Second Kirchhoff's law, the time differential equation is,

$$\frac{du_C(t)}{dt} + \frac{1}{RC}u_C(t) = \frac{1}{RC}u_{\mathrm{S}}(t) \tag{5}$$

and, taking into consideration that generator voltage is harmonic, as defined in (4),

$$u_{\mathrm{S}}(t) = \mathrm{U_{sm}}\sin(\omega t) \tag{6}$$

the solution of (5) is,

$$\begin{gathered} u_c(t) = \frac{1}{(R+R_d)C}\,\frac{\omega U_{sm}}{\omega^2 + \left[(R+R_d)C\right]^2}\cdot \\ \left[e^{-\frac{t}{RC}} - \cos(\omega t) - \frac{1}{\omega(R+R_d)C}\sin(\omega t)\right] \end{gathered} \tag{7}$$

In (7) the resistance of the diode is denoted as $\mathrm{R_d}$ and it can take the values either $\mathrm{R_{dfor}}$ or $\mathrm{R_{dinv}}$, depending on the diode status; conducting or not conducting.

This solution, denoted as PARANSOL (PARtial ANalytical SOLution) is shown in Fig. 3, together with the results of fourth order Runge – Kutta method and the "Simulink" results.

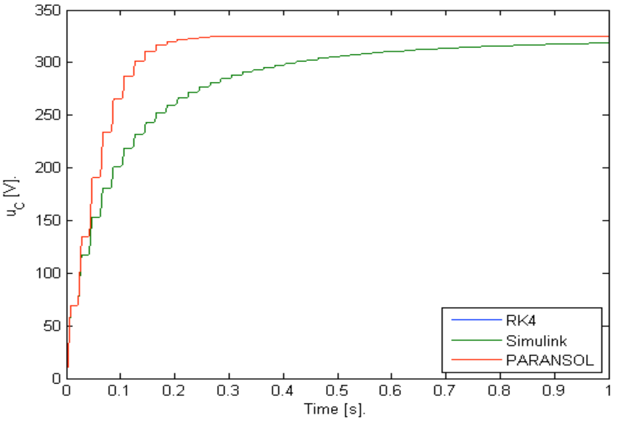

Fig. 3. *Simulation results applying fourth order of RK method, "Simulink" and "PARANSOL".*

Comparing the results presented in Fig. 3, it can be concluded that the model described by (5), with the solution (7) is much worse simulation model, comparing to all other methods presented earlier.

Two more models were investigated in order to verify previously described methods. In both cases, due to the chosen operating conditions, some bigger deviations were expected.

First, in the circuit presented in Fig. 1 a time constant voltage generator, with the voltage of $\mathrm{U}_S = \sqrt{2}\cdot 230\ \mathrm{V}$ was assumed. In the case with no diode, the circuit transient process is described by well known time differential equation,

$$\frac{du_C(t)}{dt} + \frac{1}{(R+R_d)C}u_C(t) = U_S, \tag{8}$$

during the charging of the capacitor and taking into account the diode resistance possible values, $\mathrm{R_{dfor}}$ or $\mathrm{R_{dinv}}$.

The solution of (8) is also well known,

$$u_C(t) = U_S\cdot\left(1 - \mathrm{e}^{-\frac{t}{(R+R_d)C}}\right). \tag{9}$$

During the discharging of the capacitor, the solution is well known as well,

$$u_C(t) = U_S\cdot \mathrm{e}^{-\frac{t}{(R+R_d)C}}. \tag{10}$$

In the same time a harmonic voltage, $u_s(t)$, defined by (6) was involved, defining the beginning and the end of charging and discharging processes. Namely, the end of charging capacitor process, described by (9), will occur when the capacitor voltage reaches the instantaneous voltage value, $u_s(t)$ and in the same moment starts the discharging process, presented by (10). Considering the extremely high resistance $\mathrm{R_{dinv}}$ it can be expected that the capacitor will not be discharged during the non-conducting diode state, although the generator could be treated as the ideal one, having the internal resistance equal to zero.

The capacitor's charging process will continue again when the instantaneous voltage value, $u_s(t)$ becomes higher than the capacitor' voltage.

There are two possibilities to determine the capacitor's voltage, under described circumstances. The first one is to solve the time differential equation (8), applying Runge – Kutta methods and the other possibility is to apply directly the solutions (9) and (10). Both cases were calculated and the results, together with the results of "Simulink" as the reference values, as shown in Fig. 4.

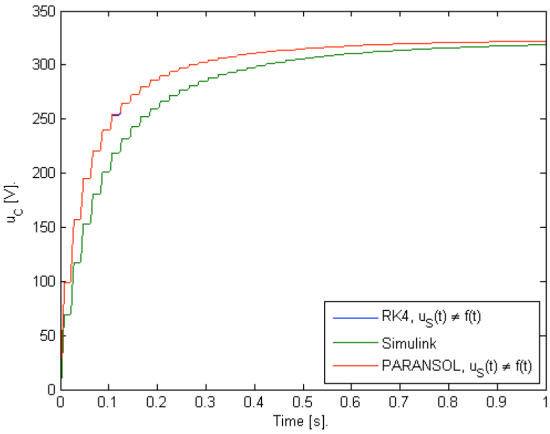

Fig. 4. *Simulation results applying fourth order of RK method and "PARANSOL" for time constant voltage.*

In Fig. 4 the case of fourth order of Runge – Kutta method, applied on time constant supplying voltage is denoted with the blue line, the application of the direct solutions (9) and (10) is presented as PARANSOL, while the reference values, obtained by "Simulink", are green.

As expected, both solutions based on time constant supply voltage are not the good approximations. The two time constant voltage approaches give very similar, but not accurate results.

In order to estimate the deviations between different approaches, the percentual deviation for each approach of capacitor voltage instantaneous values is calculated as,

$$\Delta u_c(t) = \frac{u_{capp}(t) - u_{cSimulink}(t)}{u_{cSimulink}(t)} \cdot 100[\%]. \tag{11}$$

These values, for all investigated methods whose results are presented in Fig. 2, are shown in Fig. 5.

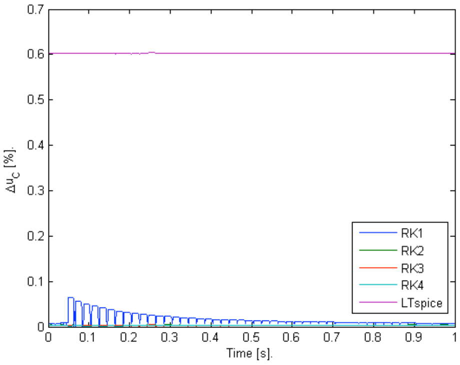

Fig. 5. *Percentual deviations of all four orders of RK method and "LTSpice" compared to the "Simulink".*

Two very improtant conclusion could be now derived from Fig. 5.

First, except the first order of explicit Runge – Kutta method, all three higher orders give almost the same results as the »Simulink« method, meaning that all explicit Runge - Kutta higher orders are as accurate as the »Simulink« method.

Second, the concept of the "LTspice" method is to operate with real elements and not with the ideal ones. For this reason, the diode investigated in "LTspice" is the real silicon one, having a forward threshold voltage of 0.6 V to 0.7 V and this is the constant, unchanged deviation, during the entire process, obvious in Fig. 5.

The percentual deviation, (11), was calculated also for the methods whose results are obtained as a solution of (5) and shown in Fig. 3. Those results are resented graphically in Fig. 6.

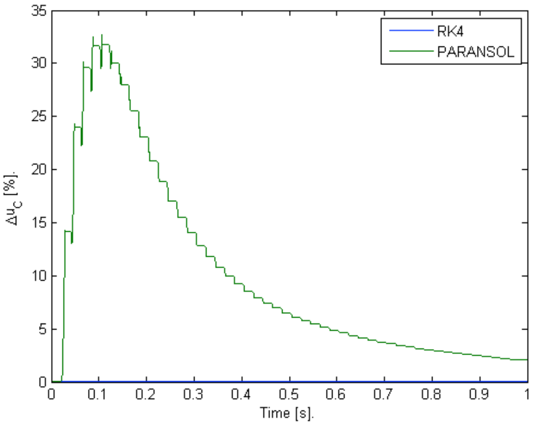

Fig. 6. *Percentual deviations of 4th order RK method and "PARANSOL", compared to the "Simulink".*

As in Fig. 3, it can be noticed a relatively small percentual deviation in the beginning of the time dependence, the huge increase of the deviation in the first ten percent of observing time, reaching the values of approximately 33% and its rapid decrease after that. As concluded earlier, this approach does not give the stable and accurate results during the entire time domain.

Finally, the percentual deviation, defined by (11) was calculated also for the two approaches with the time constant supplying voltage, as presented in (8) – (10). Fourth order Runge – Kutta method was applied to solve the differential equation (8), while the partial analytical solutions are obtained via (9) and (10). The results are presented in Fig. 7.

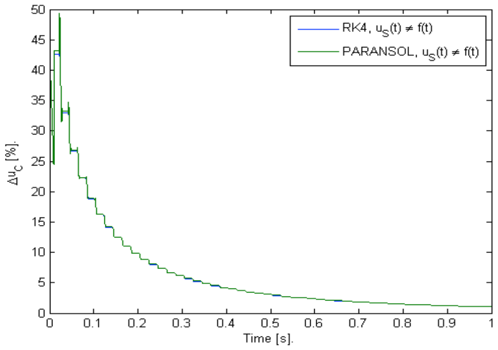

Fig. 7. *Percentual deviations of 4th order RK method and "PARANSOL", for time constant supplying voltage, compared to the "Simulink".*

As expected and as concluded earlier, those two approaches are the worst of all investigated ones. At the beginning of initial time the percentual deviation reaches almost 50 %, decreasing to very small values in the rest of observing time.

After exploring the behavior of four orders explicit Runge – Kutta method and comparing the results with four other approaches, the final conclusion can be defined. The best criterion for estimate the most appropriate procedure is the comparison of percentual capacitor voltage deviations. Maximal percentual voltage deviations are presented in Table 1.

Table 1. *Maximal percentual voltage deviations.*

Applied method	Max. Δu_c (t) [%]
1st order RK	0.0647
2nd order RK	0.0045
3rd order RK	0.0042
4th order RK	0.0035
LTspice	0.6038
4th order RK, u_s =f(t)	32.6688
PARANSOL u_s =f(t)	32.6690
4th order RK, U_s = const.	49.3923
PARANSOL, U_s = const.	49.3925

The results presented in Table 1 show more precisely that practically all orders of explicit Runge – Kutta method give the same results as the “Simulink” method and very close to the application of “LTspice”. The results obtained by four other methods, developed for the accuracy and stability verifications are much worse and, consequently, these methods could not be applied for the exact capacitor voltage determination.

Graphical presentation of all four orders of explicit Runge – Kutta method is given in Fig. 8.

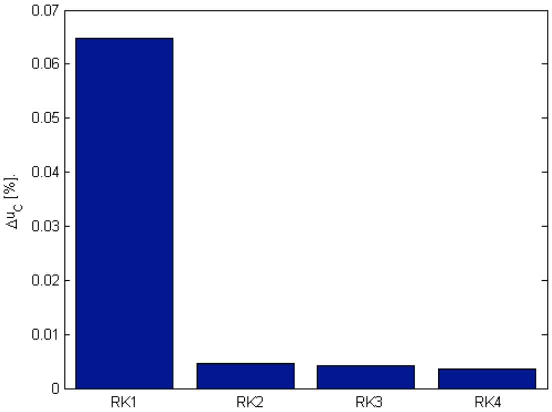

Fig. 8. *Maximal percentual deviations of four orders of Runge - Kutta method, compared to the “Simulink”.*

As already shown in Table 1, except the first order of Runge – Kutta method, with the percentual deviation less than 0.1 %, all higher orders enable even more accurate solutions of the investigated problem.

The rest of the Table 1 contents, the maximal percentual capacitor voltage deviations for other investigated approaches, are given graphically in Fig. 9.

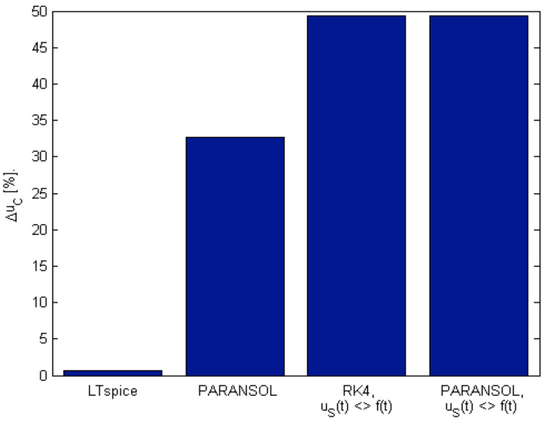

Fig. 9. *Maximal percentual deviations of other investigated approaches, compared to the “Simulink”.*

As discussed before, for, while the rest of the “LTspice” results are still accurate enough, especially for the circuits with the real elements, while the other methods are not appropriate for this kind of electrical circuits’ analysis.

4. CONCLUSION

Four different explicit Runge – Kutta methods have been analyzed and compared with the results of five other methods. The comparison show that the results obtained by all four orders of Runge – Kutta method are as accurate as the results obtained by “Simulink”, a little better from the results calculated via LTSpice and much more precise comparing to the four other approaches.

6. REFERENCES

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