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HYBRID CONTROL SCHEME FOR THE SYNCHRONIZATINO OF A TWO CHAOTIC DUFFING SCILLATOR SYSTEM SUBJECT TO UNCERTAINTIES AND EXTERNAL DISTURBANCES

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Abstract: *This paper proposes a hybrid control proposal for the synchronization of a coupled chaotic Duffing oscillator system, with uncertainties and external disturbances. Linear Quadratic Regulation (LQR) control, Sliding Mode (SM) control and Gaussian Radial basis Function Neural Network ($GRBF_{NN}$) control are combined to synchronize a chaotic phenomenon in presence of external disturbances. By Lyapunov stability theory, SM control is presented to ensure the stability of the controlled system. $GRBF_{NN}$ control is trained during the control process. The learning algorithm of the $GRBF_{NN}$ is based on the minimization of a cost function which considers both the sliding surface and control effort. Simulation results demonstrate the ability of the hybrid control scheme to synchronize the chaotic Duffing oscillator systems through the usage of a single control signal.*

KeyWords: ***Synchronization; Chaos; Linear quadratic regulation control; Sliding mode control; Neural network control; Hybrid control.***

1. INTRODUCTION

Dynamic chaos is a very interesting nonlinear effect which has been intensively studied during the last three decades. Chaotic phenomena can be found in many scientific and engineering fields such as biological systems, electronic circuits, power converters, chemical systems, and so on [1].

Since the synchronization of chaotic dynamical systems has been observed by Pecora and Carroll [2] in 1990, chaos synchronization has become a topic of great interest [3-5]. Synchronization phenomena have been reported in the recent literature. Until now, different types of synchronization have been found in interacting chaotic systems, such as complete synchronization, generalized synchronization, phase synchronization and anti-phase synchronization [6-8], etc.

Recently many researchers worked on chaos synchronization by considering Duffing oscillator problem. A.N. Njah and U.E. Vincent [9] presented chaos synchronization between single and double wells Duffing oscillators with U4 potential based on the active control technique. They used numerical simulations too present and verify the analytical results. Dibakar Ghosh, et. al presented a detailed investigation performed about the various zones of stability for delayed Duffing oscillator system, which in term shows the specific role of delay in the formation of the attractor [10]. In the study of phase synchronization the machinery of empirical mode decomposition analysis is adapted and lastly maximal Lyapunov exponent is computed as a verifying criterion. Rene Yamapi and Giovanni Filatrella [11] considered the synchronization dynamics of coupled chaotic Duffing oscillator systems.

There are many other recent reports on the synchronization of Duffing oscillator oscillators. H.G. Enjieu Kadjia and R. Yamapi [12] considered the general synchronization dynamics of coupled Duffing oscillators and examined the linear and nonlinear stability analysis on the synchronization process through the Whittaker method and the Floquet theory in addition to the multiple time scales method. Our work mainly focused on the systems considered in [12].

In this study, a hybrid control scheme is applied to chaos synchronization. Two identical chaotic systems such as Duffing oscillator have been considered as the master and the slave systems. The slave system has been subjected to model uncertainty and external disturbances. To achieve the presented goal, some control techniques such as LQR, SMC, GRBFNN and hybrid control have been designed.

This paper is organized as follows. In section II, the dynamics of a nonlinear duffing system is explained. The synchronization problem for nonlinear duffing systems is described in section III. In section IV, the material and methods are explained. In this section, LQR control is designed. Also, SM control is designed. GRBF$_{NN}$ control and the learning algorithms of this controller are presented. Moreover, hybrid control for synchronizing of nonlinear duffing systems is presented. Finally, to show the effectiveness of these control methods for synchronization, simulations are presented in section V. At the end, the paper is concluded in section VI.

2. DUFFING OSCILLATOR SYSTEM

Consider a second-order chaotic system such as well known Duffing's equation describing a special nonlinear circuit or a pendulum moving in a viscous medium under control [13]:

$$\ddot{x} = -p\dot{x} - p_1 x - p_2 x^3 + q\cos(\omega t) \quad (1)$$

where p , p_1 , p_2 and q are real constants . t is the time variable and ω is the frequency.

Given the states $x_1 = x$ and $x_2 = \dot{x}$, then the Eq. (1) can be rewritten as follow:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -px_2 - p_1 x_1 - p_2 x_1^3 + q\cos(\omega t) \end{cases} \quad (2)$$

This system exhibits complex dynamics and has been studied by [13]. The constant values of Eq. (2) are $p = 0.4$, $p_1 = -1.1$, $p_2 = 1$, $q = 0.62$ and $\omega = 1.8$.

Fig. 1 and Fig. 2 illustrate the irregular motion exhibited by this system and initial conditions of $(x_1, x_2) = (1, -1)$.

In the next section, the problem of synchronizing two identical Duffing system with different initial conditions is described.

Notice that model uncertainty and external disturbances appear in the slave system.

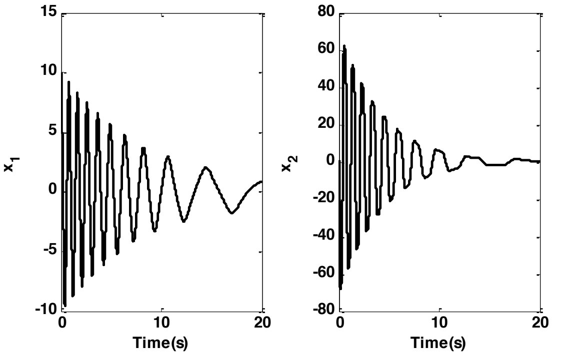

Fig.1. *Time series of* x_1 *and* x_2

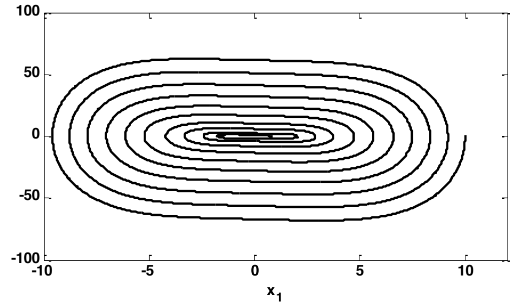

Fig. 2. *Phase plane trajectory of a chaotic Duffing oscillator system*

3. SYNCHRONIZATION PROBLEM

Consider two coupled, chaotic gyro systems are as following:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -px_2 - p_1 x_1 - p_2 x_1^3 + q\cos(\omega t) = g(x_1, x_2, \omega, t) \end{cases} \quad (3)$$

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = -py_2 - p_1 y_1 - p_2 y_1^3 + q\cos(\omega t) + \Delta f(y_1, y_2) + d(t) + u(t) \\ \quad = g(y_1, y_2, \omega, t) + \Delta f(y_1, y_2) + d(t) + u(t) \end{cases} \quad (4)$$

where $u \in R$ is the control input, $\Delta f(y_1, y_2)$ is an uncertainty term representing the un-modeled dynamics or structural variation of the system is given in Eq. (4) and $d(t)$ is the time-varying disturbance.

In general, the uncertainty and the disturbance are assumed to be bounded as follows:

$$|\Delta f(y_1, y_2)| \le \alpha \text{ And } |d(t)| \le \beta .$$

where α and β are positive constant values.

The systems described in Eq. (3) and Eq. (4) correspond to the master system and the slave system, respectively, and the objective of the current control problem is to design an appropriate control signal $u(t)$ such that for any initial conditions of the two systems, the behavior of the slave converges to that of the master. Defining the state errors between the master and slave systems as:

$$\begin{cases} e_1 = y_1 - x_1 \\ e_2 = y_2 - x_2 \end{cases} \quad (5)$$

Then, the dynamics equations of these errors can be determined by subtracting Eq. (3) from Eq. (4) as follow:

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = g(y_1, y_2, \omega, t) - g(x_1, x_2, \omega, t) \\ \qquad + \Delta f(y_1, y_2) + d(t) + u(t) \end{cases} \quad (6)$$

4. MATERIAL AND METHODS

A. Linear Quadratic Regulation Control

This method determines the state feedback gain matrix that minimizes J in order to achieve some compromise between the use of control effort, the magnitude, and the speed of response that together guarantee a stable system. After linearization of the system:

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) \tag{7}$$

Determine the control effort $u(t)$ as:

$$u(t) = -kx(t) \tag{8}$$

And $k = R^{-1}B^T X$, where X is obtained from Matrix Differential Riccati Equation (MDRE), MDRE is as follows:

$$\begin{aligned} -\dot{X}(t) &= A^T(t)X(t) + X(t)A^T(t) \\ &+Q(t) - X(t)B(t)R^{-1}B^T(t)X(t) \end{aligned} \tag{9}$$

So in order to minimize the performance index,

$$J = \frac{1}{2}\int_0^\infty (e^T Qe + u^T Ru)dt \tag{10}$$

where Q and R are the positive definite Hermitian or real symmetric matrices.

Note that the second term on the right hand side accounts for the expenditure of the energy on the control efforts, the matrix of Q and R determine the relative importance of the error and the expenditure of this energy [14].

B. Sliding surface and sliding mode control

Using an SM control method to synchronize the chaotic Duffing oscillator system, involves two basic steps;

(1) Selecting an appropriate sliding surface such that the sliding motion on the sliding manifold is stable.

(2) Establishing a robust control law which guarantees the existence of the sliding manifold $S(t) = 0$ even in the event of uncertainties. The sliding surface is defined as [15]:

$$S(t) = e_2(t) + \delta\, e_1(t) \tag{11}$$

where δ is a real positive constant. The rate of convergence of the sliding surface is governed by the value assigned to parameter δ. The first derivative of (10) with respect to time is:

$$\dot{S}(t) = \dot{e}_2(t) + \delta\, \dot{e}_1(t) \tag{12}$$

Substituting the Eq. (6) into Eq. (12):

$$\begin{aligned} \dot{S}(t) &= g(y_1, y_2, \omega, t) - g(x_1, x_2, \omega, t) + \Delta f(y_1, y_2) \\ &+d(t) + u(t) + \delta\, e_2(t) \end{aligned} \tag{13}$$

Define a Lyapunov function as:

$$V = \frac{1}{2}S^2 \tag{14}$$

Differentiating Eq. (14) with respect to time we have:

$$\dot{V} = S\dot{S} \tag{15}$$

Substituting Eq. (13) into (15):

$$\begin{aligned} \dot{V} &= S\left[g(y_1, y_2, \omega, t) - g(x_1, x_2, \omega, t)\right] \\ &+S\left[\Delta f(y_1, y_2) + d(t) + u(t) + \delta\, e_2(t)\right] \end{aligned} \tag{16}$$

Let

$$u(t) = -\eta\,\mathrm{sgn}(S) + g(x_1, x_2, \omega, t) - g(y_1, y_2, \omega, t) - \delta\, e_2(t)$$
(17)

where η is a positive constant and $\eta > \alpha + \beta$. Then

$$\dot{V} = -\eta|S| \tag{18}$$

Since $\eta > \alpha + \beta$, the reaching condition ($S\dot{S} < 0$) is always satisfied. Thus, the proof is achieved. An appropriate value of η is chosen not only to quicken the time of reaching the sliding mode motion which has a good robustness to the system uncertainties, but also to reduce the system chattering. Therefore, this implies that the sliding surface be chattering in a finite time and the SM controller is used for synchronizing the chaotic Duffing oscillator systems. Thus the error state trajectories converge to the sliding surface $S(t) = 0$.

C. GRBF Neural Network

The $\mathrm{GRBF}_{\mathrm{NN}}$ can be considered as one layer feed forward neural network with nonlinear element. The $\mathrm{GRBF}_{\mathrm{NN}}$ output can perform the mapping according to:

$$f(z) = \sum_{j=1}^{n} w_j G_j(z_j, m_j, \sigma_j) \tag{19}$$

where $z = [z_1, z_2, \ldots, z_n]^T \in R^n$ is the input vector, $G_j(z_j, m_j, \sigma_j) \in R^n$, $j = 1,2,\ldots,n$ are the Gaussian radial basis functions, m_j is the mean value of the Gaussian function, $\sigma_j \in R$ is the spread of Gaussian function n is the number of neurons.

Each Gaussian radial basis function can be represented by:

$$G_j(z_j, m_j, \sigma_j) = \exp\left(\frac{z_j - m_j}{\sqrt{2}\sigma_j}\right)^2 \tag{20}$$

$\mathrm{GRBF}_{\mathrm{NN}}$ can be used for synchronizing the chaotic Duffing oscillator systems. To achieve this goal, it is assumed that the output of $\mathrm{GRBF}_{\mathrm{NN}}$ is the control effort of the synchronization, then $u(t) = f$.

D. Learning Algorithm of GRBFNN

The goal is to minimize the following cost function:

$$E(k) = S(k)\dot{S}(k) + \frac{1}{2}(u^T(k)u(k)) \tag{21}$$

where $S(k)$ is the sliding surface that was described in the previous section. By using the BP algorithm, the weighting vector of the GRBF$_{NN}$ is adjusted such that the cost function defined in Eq. (21) is less than designed. The well-known algorithm may be written briefly as:

$$w(k+1) = w(k) + \gamma\left(-\frac{\partial E(k)}{\partial w}\right) \quad (22)$$

where γ and w represent the learning rate and tuning parameter of RBFNN. The gradient of $E(.)$ in Eq. (22) with respect to a weighting w is:

$$\frac{\partial E(k)}{w} = S(k)\frac{\partial \dot{S}(k)}{\partial f}\frac{\partial f(k)}{\partial w} + u(k)\frac{\partial f(k)}{\partial w} \quad (23)$$

where

$\frac{\partial f(k)}{\partial w} = G(z,m,\sigma)$ and $\frac{\partial \dot{S}(k)}{\partial f} = 1$.Then,

$$\frac{\partial E(k)}{w} = (S(k) + u(k))G(z,m,\sigma) \quad (24)$$

Substituting the Eq. (24) into the (22):

$$w(k+1) = w(k) + \gamma(S(k) + u(k))G(z,m,\sigma) \quad (25)$$

E. Hybrid Control

The structure of hybrid control to synchronize the chaotic Duffing oscillator system is shown in Fig. 3.The total control effort is computed as follows:

$$u(t) = u_{LQR}(t) + m(t)u_{SM}(t) + (1-m(t))u_{GRBF_{NN}}(t) \quad (26)$$

where $u_{LQR}(t)$ is the LQR control, $u_{SM}(t)$ is the SM control and $u_{GRBF_{NN}}(t)$ is the GRBF$_{NN}$ control. The function $m(t)$ allows a smooth transition between the SM controller and the GRBF$_{NN}$ controller, based on the location of the system state:

$$\begin{cases} m(t) = 0 & e(t) \in A_d \\ 0 < m(t) < 1 & otherwise \\ m(t) = 1 & e(t) \in A_c \end{cases} \quad (27)$$

where the regions might be defined as in Fig. 4.

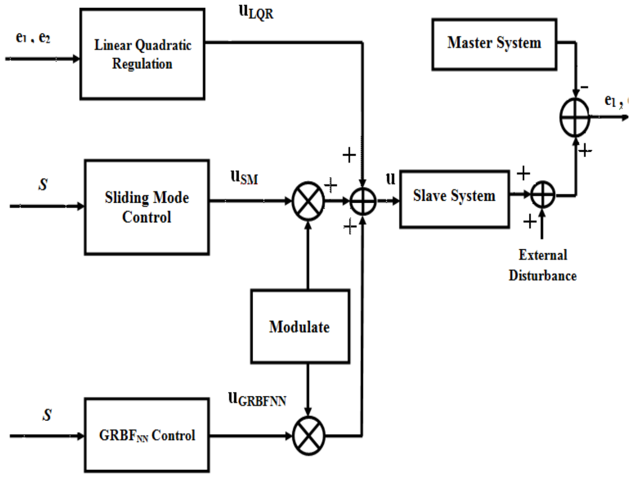

Fig. 3. *Structure of hybrid control to synchronize of chaotic Duffing oscillator system*

The SM controller is used to keep the error states in a region where the neural network can be accurately trained to achieve optimal control. The SM controller is turned on (and the neural controller is turned off) whenever the system error states drifts outside this region. The combination of controllers produces a stable system, which adapts to optimize performance.

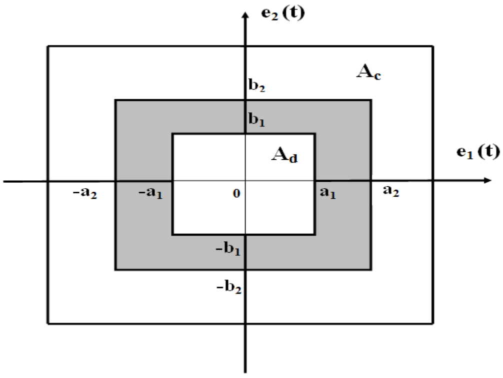

Fig. 4. *Controller regions*

5. SIMULATION RESULTS

The parameters of chaotic Duffing oscillator systems are specified as follows:

$p = 0.4$, $p_1 = -1.1$, $p_2 = 1$, $q = 0.62$ and $\omega = 1.8$, which, as shown in section 2, give rise to a chaotic state.

The initial conditions are defined as:

$x_1(0) = 10$, $x_2(0) = 1$, $y_1(0) = 5$, $y_2(0) = 11$.

Also, an assumption is made that the uncertainty term, $\Delta f(y_1, y_2) = -\sin(y_1)$ and the disturbance term, $d(t) = rand$ are bounded by $|\Delta f(y_1, y_2)| \le \alpha = 1$ and $|d(t)| \le \beta = 1$, respectively.

The simulation results are shown in Figures 5-17.

Figs. 5, 8, 11 and Fig. 14 show time series of the master and slave states corresponding to their control methods. Figs. 6, 9, 12 and Fig. 15 show time series of synchronization errors corresponding to their control methods. Moreover, the performance index *I* is defined as $I = \sqrt{e_1^2 + e_2^2}$. The performance index *I* is shown that the proposed hybrid control method can achieve favorable tracking performance. This Index is chosen as the optimality criteria in this study. For all control methods applied to Duffing system, their indexes are shown in Fig.18.

The simulation results of hybrid control have a good performance in comparison to other control methods that were applied in this section. The simulation results of hybrid control confirm that the master and the slave systems achieve the synchronized states before 1 sec. Also, these results demonstrate that the system error states are regulated to zero asymptotically before 1 sec.

In addition, it can be seen that the results of hybrid control have a good performance even though the overall system is subject to uncertainty and disturbance. The proposed hybrid method is very efficient and powerful to synchronize Duffing systems in compare of [9]. Also, in this study, the synchronization goal is achieved faster than [9].

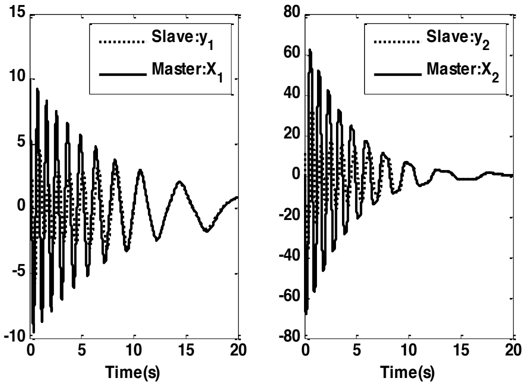

Fig. 5. *Time series of the master and slave states with LQR control*

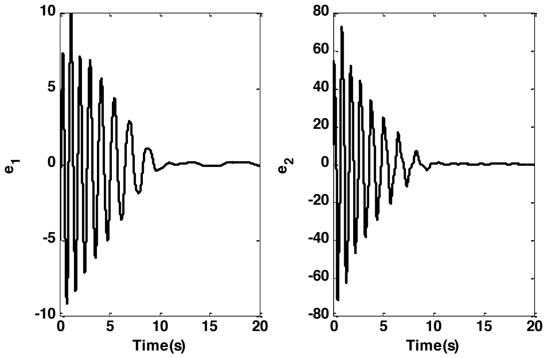

Fig. 6. *Time series of synchronization errors with LQR control*

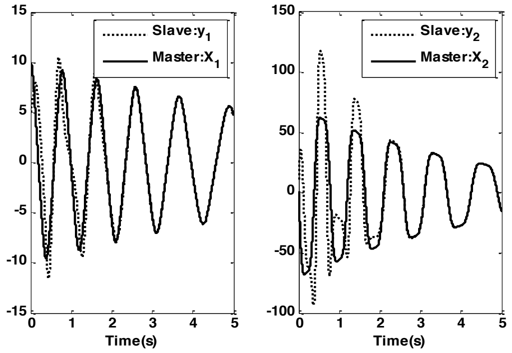

Fig. 7. *Time series of the master and slave states with SM control*

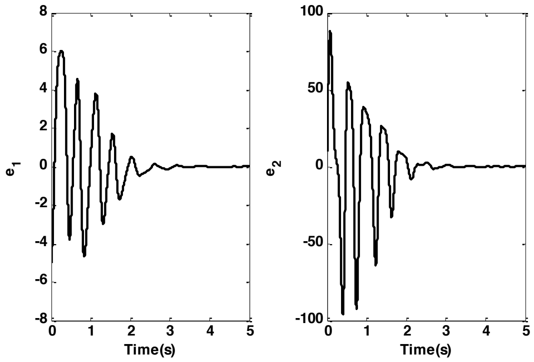

Fig. 8. *Time series of synchronization errors with SM control.*

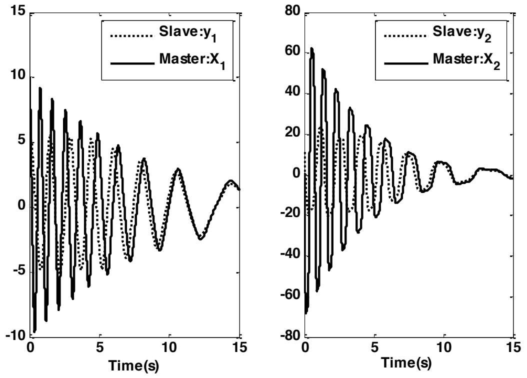

Fig. 9. *Time series of the master and slave states with* $GRBF_{NN}$ *control*

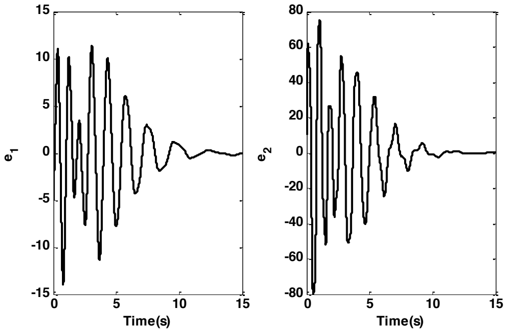

Fig. 10. *Time series of synchronization errors with* $GRBF_{NN}$ *control*

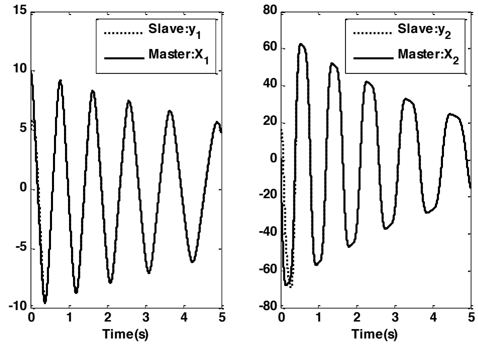

Fig. 11. *Time series of the master and slave states with hybrid control*

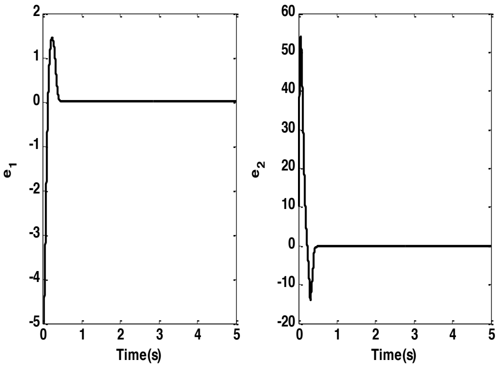

Fig. 12. *Time series of synchronization errors with hybrid control*

The problem of controller complexity is a very crucial issue in the practical implementation [16], for example in electronics and engineering applications. Two fundamental issues in this direction are (a) the cost implication and density requirement for designing controllers and (b) the need to make the complexity of the controller to be comparable with, or less than, the device being controlled, if the controlling technique is desired to achieve a useful end and not merely a scientific curiosity.

6. CONCLUSION

This paper presents a hybrid control scheme for the synchronization of chaotic Duffing oscillator systems characterized by system uncertainties and disturbances. This hybrid control scheme is highly robust and achieves a stable, controlled system despite the presence of uncertainties and disturbances. Simulation results of hybrid control have a good performance in comparison to other control methods applied in this paper. According to these simulations, the proposed hybrid method can be successfully applied to the synchronization problem of chaotic Duffing oscillator systems.

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